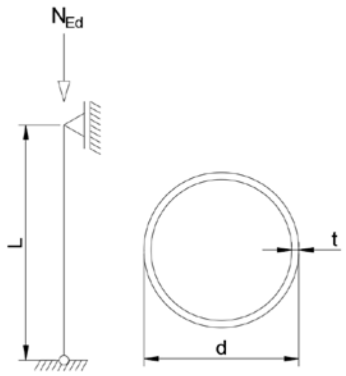
Assignment 1



Properties

Cold-formed CHS 159 × 4, cold-rolled strip, Austenitic grade 1.4307

$L = 3{,}5\ m$	$A = 19{,}5\ cm^2$
$N_{Ed} = 250\ kN$	$I = 585{,}3\ cm^4$
$d = 159\ mm$	$W_{el} = 73{,}6\ cm^3$
$t = 4\ mm$	$W_{pl} = 96{,}1\ cm^3$

Goal

- Draw the NVM diagram’s for this example
- Conduct the calculation of this structure using the design manual,
- Conduct the cross section resistance calculation of this structure using the enhanced average yield strength and the continuous strength method,
- Use the application on an iPad to calculate the example.

Reflection

- Have you checked your results with the correction? How did you do? Where did it go wrong? Do you understand the mistake?
- Do you notice a large difference in results taking into account enhanced average yield strength and CSM? What is the difference in % using the first calculation as a reference for both enhanced average yield strength and CSM?
- When you recalculate the example using the software application. Do the results match your calculations? Why/why not? If not, what is the reason of the difference?

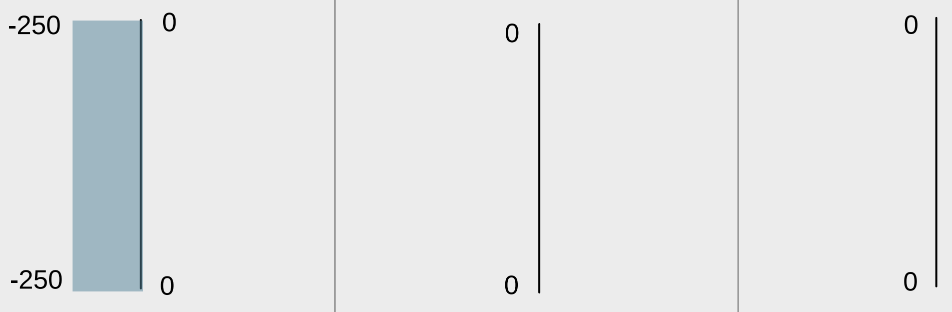
Results

$N_{\mathrm{c,Rd}} = 390\ kN$, $N_{\mathrm{b,Rd}} = 288{,}6\ kN$, $f_{ya} = 245\ N/mm^2$, $f_{csm} = 266\ N/mm^2$, $N_{\mathrm{csm,Rd,}} = 471{,}6\ kN$

Solution assignment 1

NVM

N [kN]	V [kN]	M [kNm]



Properties

Cold-formed CHS 159 × 4, cold-rolled strip, Austenitic grade 1.4307

$L = 3{,}5\ m$	$A = 19{,}5\ cm^2$
$N_{Ed} = 250\ kN$	$I = 585{,}3\ cm^4$
$d = 159\ mm$	$W_{el} = 73{,}6\ cm^3$
$t = 4\ mm$	$W_{pl} = 96{,}1\ cm^3$
$f_y = 220\ N/mm^2$	$E = 200000\ N/mm^2$
$f_u = 520\ N/mm^2$	

Classification of the cross-section

$\varepsilon = 1{,}01$

$d/t = 159/4 = 39{,}8$

For Class 1, $d/t \leq 50\varepsilon^2$, therefore the section is Class 1.

Compression resistance of the cross-section

For a Class 1 cross-section:

$$N_{\mathrm{c,Rd}} = A_{\mathrm{g}} f_{\mathrm{y}} / \gamma_{\mathrm{M0}}$$

$$N_{\mathrm{c,Rd}} = \frac{19{,}5 \times 220 \times 10^{-1}}{1{,}1} = 390\ \mathrm{kN}$$

Resistance to flexural buckling

$$N_{\mathrm{b,Rd}} = \frac{\chi A f_{\mathrm{y}}}{\gamma_{\mathrm{M1}}}$$

$$\chi = \frac{1}{\phi + \left[\phi^2 - \bar{\lambda}^2\right]^{0,5}} \leq 1$$

$$\phi = 0,5(1 + \alpha\left(\bar{\lambda} - \overline{\lambda}_0\right) + \bar{\lambda}^2)$$

Calculate the elastic critical buckling load:

$$N_{\mathrm{cr}} = \frac{\pi^2 EI}{L_{\mathrm{cr}}{}^2} = \frac{\pi^2 \times 200000 \times 585,3 \times 10^4}{(3,50 \times 10^3)^2} \times 10^{-3} = 943,1\ \mathrm{kN}$$

Calculate the flexural buckling slenderness:

$$\bar{\lambda} = \sqrt{\frac{19,5 \times 10^2 \times 220}{943,1 \times 10^3}} = 0,67$$

Using an imperfection factor $\alpha = 0,49$ and $\bar{\lambda}_0 = 0,2$ for a cold-formed austenitic stainless steel CHS:

$$\phi = 0,5 \times (1 + 0,49 \times (0,67 - 0,2) + 0,67^2) = 0,84$$

$$\chi = \frac{1}{0,84 + [0,84^2 - 0,67^2]^{0,5}} = 0,74 \leq 1$$

$$N_{\mathrm{b,Rd}} = 0,74 \times 19,5 \times 220 \times \frac{10^{-1}}{1,1} = 288,6\ \mathrm{kN}$$

The applied axial load is $N_{\mathrm{Ed}} = 250$ kN.

Thus the member has adequate resistance to flexural buckling.

Strength enhancement of cold formed sections

$$f_{\mathrm{ya}} = f_{\mathrm{yCHS}} = 0,85K\left(\varepsilon_{\mathrm{CHS}} + \varepsilon_{\mathrm{p0,2}}\right)^{n_{\mathrm{p}}}\ and\ f_{\mathrm{y}} \leq f_{\mathrm{yCHS}} \leq f_{\mathrm{u}}$$

Calculate various parameters to continue the calculation:

$$\varepsilon_{\mathrm{CHS}} = \frac{t}{2(d-t)} = \frac{4}{2(159-4)} = 0,0129$$

$$\varepsilon_{\mathrm{p0,2}} = 0,002 + \frac{f_{\mathrm{y}}}{E} = 0,002 + \frac{220}{200\,000} = 0,0031$$

$$\varepsilon_{\mathrm{u}} = 1 - \frac{f_{\mathrm{y}}}{f_{\mathrm{u}}} = 1 - \frac{220}{520} = 0,5769$$

$$n_{\mathrm{p}} = \frac{ln(f_{\mathrm{y}}/f_{\mathrm{u}})}{ln(\varepsilon_{\mathrm{p0,2}}/\varepsilon_{\mathrm{u}})} = \frac{ln(220/520)}{ln(0,0031/0,5769)} = 0,1646$$

$$K = \frac{f_{\mathrm{y}}}{\varepsilon_{\mathrm{p0,2}}^{n_{\mathrm{p}}}} = \frac{220}{0,0031^{0,1646}} = 569,30$$

Thus:

$$f_{\mathrm{ya}} = 0,85 \cdot 569,30\ (0,0129 + 0,0031)^{0,1646} = 245\ N/mm^2$$

$$220\ N/mm^2 \leq 245\ N/mm^2 \leq 520\ N/mm^2$$

Boundaries are good.

Continuous Strength Method (CSM)

For this calculation $f_{\mathrm{y}} = f_{\mathrm{ya}} = 245\ N/mm^2$.

$$\varepsilon_{\mathrm{y}} = f_{\mathrm{y}}/E = 245/200\,000 = 0,001225$$

$$\varepsilon_{\mathrm{u}} = C_3\left(1 - f_{\mathrm{y}}/f_{\mathrm{u}}\right) = 1{,}00 \cdot (1 - 245/520) = 0{,}529$$

Table D.1	*Stainless steel*	*C_1*	*C_2*	*C_3*	
	Austenitic	*0,10*	*0,16*	*1,00*	

$$E_{\mathrm{sh}} = \frac{f_{\mathrm{u}} - f_{\mathrm{y}}}{C_2\varepsilon_{\mathrm{u}} - \varepsilon_{\mathrm{y}}} = \frac{520 - 245}{0{,}16 \cdot 0{,}529 - 0{,}001225} = 3296{,}77\ N/mm^2$$

$$\frac{\varepsilon_{\mathrm{csm}}}{\varepsilon_{\mathrm{y}}} = \begin{cases} \dfrac{4{,}44 \times 10^{-3}}{\bar{\lambda}_{\mathrm{c}}^{\,4{,}5}} \ \leq \ \min\left(15; \dfrac{C_1\varepsilon_{\mathrm{u}}}{\varepsilon_{\mathrm{y}}}\right) & \text{for } \bar{\lambda}_{\mathrm{c}} \leq 0{,}30 \\ \left(1 - \dfrac{0{,}224}{\bar{\lambda}_{\mathrm{c}}^{\,0{,}342}}\right)\dfrac{1}{\bar{\lambda}_{\mathrm{c}}^{\,0{,}342}} & \text{for } \bar{\lambda}_{\mathrm{c}} > 0{,}30 \end{cases}$$

First calculate $\bar{\lambda}_{\mathrm{c}}$:

$$\bar{\lambda}_{\mathrm{c}} = \sqrt{f_{\mathrm{y}}/f_{\mathrm{cr,c}}} = \sqrt{245/6090{,}34} = 0{,}20 \leq 0{,}30$$

$$f_{\mathrm{cr,c}} = \frac{E}{\sqrt{3(1-\upsilon^2)}}\frac{2t}{D} = \frac{200\ 000}{\sqrt{3(1-0{,}3^2)}}\frac{2 \cdot 4}{159} = 6090{,}34\ N/mm^2$$

$\bar{\lambda}_{\mathrm{c}} \leq 0{,}30$ thus:

$$\frac{\varepsilon_{\mathrm{csm}}}{\varepsilon_{\mathrm{y}}} = \frac{4{,}44 \times 10^{-3}}{0{,}2^{4{,}5}} = 6{,}21 \ \leq \ \min\left(15; \frac{0{,}1 \cdot 0{,}529}{0{,}001225}\right) = \min(15; 43{,}18)$$

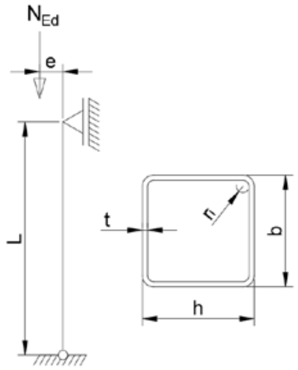
Boundaries are satisfied.

Cross section resistance

$$f_{\mathrm{csm}} = f_{\mathrm{y}} + E_{\mathrm{sh}}\varepsilon_{\mathrm{y}}\left((\varepsilon_{\mathrm{csm}}/\varepsilon_{\mathrm{y}}) - 1\right) = 245 + 3296{,}77 \cdot 0{,}001225(6{,}21 - 1) = 266\ N/mm^2$$

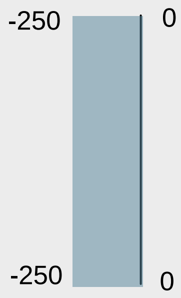
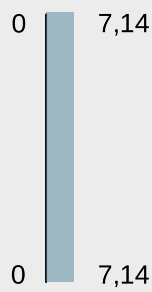
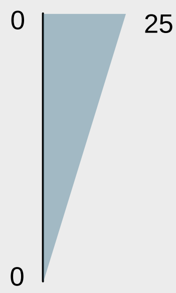
$$N_{\mathrm{c,Rd}} = N_{\mathrm{csm,Rd}} = \frac{Af_{\mathrm{csm}}}{\gamma_{\mathrm{M0}}} = \frac{1950 \cdot 266}{1{,}1} = 471{,}6\ kN$$

End of calculation.

Assignment 2	
	
Properties	
Cold-rolled square hollow section (SHS) 100x100x5, ferritic grade 1.4016.	
$L = 3{,}5\ m$	$r_i = 5\ mm$
$N_{Ed} = 250\ kN$	$A = 1818{,}45\ mm^2$
$h = 100\ mm$	$I = 266{,}79\ cm^4$
$b = 100\ mm$	$W_{el} = 53{,}36\ cm^3$
$t = 5\ mm$	$W_{pl} = 63{,}73\ cm^3$
Goal	
– Draw the NVM diagram's for this example. – Can the cross section resist a force of 250 kN applied with an eccentricity of $e = 100\ mm$? For this example only concider the ultimate resistance using the enhanced average yield strength and CSM, no buckling check is recuired.	
Reflection	
– Have you checked your results with the correction? How did you do? Where did it go wrong? Do you understand the mistake?	
Results	
$f_{ya} = 329{,}55\ N/mm^2$, $f_{cr,p} = 2501{,}90\ N/mm^2$, $M_{\mathrm{csm,y,Rd,}} = 20{,}27\ kNm$, $N_{\mathrm{csm,Rd}} = 584{,}76\ kN$, $M_{\mathrm{csm,y,Rd,}} = 15{,}10\ kNm$	

Solution assignment 2

NVM

N [kN]	V [kN]	M [kNm]
		

Properties

Cold-rolled square hollow section (SHS) 100x100x5, ferritic grade 1.4016.

$L = 3{,}5\, m$	$r_i = 5\, mm$
$N_{Ed} = 250\, kN$	$A = 1818{,}45\, mm^2$
$h = 100\, mm$	$I = 266{,}79\, cm^4$
$b = 100\, mm$	$W_{el} = 53{,}36\, cm^3$
$t = 5\, mm$	$W_{pl} = 63{,}73\, cm^3$
$f_y = 260\, N/mm^2$	$E = 200000\, N/mm^2$
$f_u = 450 N/mm^2$	

Strength enhancement of cold formed sections

$$f_{\mathrm{ya}} = \frac{f_{\mathrm{yc}}\, \mathrm{A}_{\mathrm{c,rolled}} + f_{\mathrm{yf}}(A - A_{\mathrm{c,rolled}})}{A}$$

In which:

$$f_{\mathrm{yc}} = 0{,}85K\left(\varepsilon_{\mathrm{c}} + \varepsilon_{\mathrm{p0,2}}\right)^{n_{\mathrm{p}}}\ and\ f_{\mathrm{y}} \leq f_{\mathrm{yc}} \leq f_{\mathrm{u}}$$

$$f_{\mathrm{yf}} = 0{,}85K\left(\varepsilon_{f} + \varepsilon_{\mathrm{p0,2}}\right)^{n_{\mathrm{p}}}\ and\ f_{\mathrm{y}} \leq f_{\mathrm{yf}} \leq f_{\mathrm{u}}$$

Calculate various parameters to continue the calculation:

$$\varepsilon_{\mathrm{p0,2}} = 0{,}002 + \frac{260}{200\,000} = 0{,}0033$$

$$\varepsilon_{\mathrm{c}} = \frac{t}{2(2r_i + t)} = \frac{5}{2(2 \cdot 5 + 5)} = 0{,}1667$$

$$\varepsilon_{\mathrm{f}} = \left[\frac{t}{900}\right] + \left[\frac{\pi t}{2(b + h - 2t)}\right] = \left[\frac{5}{900}\right] + \left[\frac{\pi \cdot 5}{2(100 + 100 - 2 \cdot 5)}\right] = 0{,}047$$

$$n_{\mathrm{p}} = \frac{ln(f_{\mathrm{y}}/f_{\mathrm{u}})}{ln(\varepsilon_{\mathrm{p0,2}}/\varepsilon_{\mathrm{u}})} = \frac{ln(260/450)}{ln(0{,}0033/0{,}253)} = 0{,}126$$

$$K = \frac{f_y}{\varepsilon_{p0,2}^{n_p}} = \frac{260}{0{,}0033^{0{,}126}} = 534{,}12$$

$$\varepsilon_u = 0{,}6\left[1 - \frac{f_y}{f_u}\right] = 0{,}6\left[1 - \frac{260}{450}\right] = 0{,}253$$

Calculate $A_{c,rolled}$ where n_c is the number of 90° corners in the section

$$A_{c,rolled} = \left(n_c\,\pi\frac{t}{4}\right)(2r_i + t) \;+\; 4n_c t^2 = \left(4\cdot\pi\frac{5}{4}\right)(2\cdot 5 + t) \;+\; 4\cdot 4\cdot 5^2 = 635{,}62\;mm^2$$

$$f_{yc} = 0{,}85\cdot 534{,}12\;(0{,}1667 + 0{,}0033)^{0{,}126} = 363{,}16\;N/mm^2$$

$$BUT\!: 260\;N/mm^2 \le 363{,}16\;N/mm^2 \le 450\;N/mm^2\;(OK)$$

$$f_{yf} = 0{,}85\cdot 534{,}12\;(0{,}047 + 0{,}0033)^{0{,}126} = 311{,}50\;N/mm^2$$

$$BUT\!: 260\;N/mm^2 \le 311{,}50\;N/mm^2 \le 450\;N/mm^2\;(NOT\;OK)$$

Thus f_{ya} should be calculated using $f_{yf} = 311{,}50\;N/mm^2$ and $f_{yc} = 363{,}16\;N/mm^2$:

$$f_{ya} = \frac{363{,}16\cdot 635{,}62 \;+\; 311{,}50\cdot(1818{,}45 - 635{,}62)}{1818{,}45} = 329{,}55\;N/mm^2$$

Continuous Strength Method (CSM)

For this calculation $f_y = f_{ya} = 329{,}55\;N/mm^2$.

$$\varepsilon_y = f_y/E = 329{,}55/200\,000 = 0{,}0016$$

$$\varepsilon_u = C_3\left(1 - f_y/f_u\right) = 0{,}6\cdot(1 - 329{,}55/450) = 0{,}161$$

Table D.1	*Stainless steel*	*C_1*	*C_2*	*C_3*	
	Ferritic	*0,40*	*0,45*	*0,60*	

$$E_{sh} = \frac{f_u - f_y}{C_2\varepsilon_u - \varepsilon_y} = \frac{450 - 329{,}55}{0{,}45\cdot 0{,}161 - 0{,}0016} = 1700{,}07\;N/mm^2$$

$$\frac{\varepsilon_{csm}}{\varepsilon_y} = \begin{cases} \dfrac{0{,}25}{\bar{\lambda}_p^{\;3{,}6}} \;\le\; \min\left(15, \dfrac{C_1\varepsilon_u}{\varepsilon_y}\right) & \text{for } \bar{\lambda}_p \le 0{,}68 \\ \left(1 - \dfrac{0{,}222}{\bar{\lambda}_p^{\;1{,}050}}\right)\dfrac{1}{\bar{\lambda}_p^{\;1{,}050}} & \text{for } \bar{\lambda}_p > 0{,}68 \end{cases}$$

First calculate $\bar{\lambda}_p$:

$$\bar{\lambda}_p = \sqrt{f_y/f_{cr,p}}$$

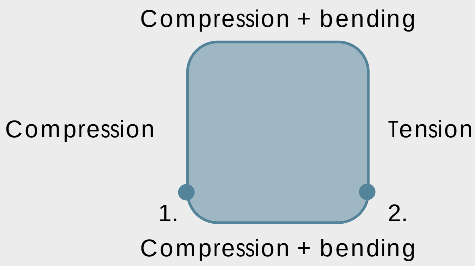
$$f_{cr,p} = \frac{k_\sigma \pi^2 E t^2}{12(1 - \upsilon^2)\bar{b}^2}$$

In which:

$$\bar{b} = b - 3t = 100 - 3 \cdot 5 = 85\ mm$$

$$\upsilon = 0{,}3$$

Since this is a compression & bending situation you need to calculate the stresses caused in the profile. You need them to determine k_σ. See table 5.3 and 5.4 in the design manual.



However, on the drawing you can see a sides completely in compression and in combination compression with bending. We know that the compression zone will give more conservative results. Therefore we should not calculate $f_{\mathrm{cr,p}}(compression\ \&\ bending)$. To be complete we included the calculation as well.

$$\sigma_1 = \frac{N}{A} + \frac{My}{I} = \frac{250\ 000}{1818{,}45} + \frac{25 \cdot 10^6 \cdot 50}{266{,}79 \cdot 10^4} = 606\ N/mm^2 (compression)$$

$$\sigma_2 = \frac{N}{A} - \frac{My}{I} = \frac{250\ 000}{1818{,}45} - \frac{25 \cdot 10^6 \cdot 50}{266{,}79 \cdot 10^4} = 331\ N/mm^2 (tension)$$

$$\psi = \frac{\sigma_2}{\sigma_1} = \frac{331}{-606} = -0{,}55$$

$$k_\sigma = 7{,}81 - 6{,}29\psi + 9{,}78\psi^2 = 7{,}81 - 6{,}29 \cdot (-0{,}55) + 9{,}78 \cdot (-0{,}55)^2 = 14{,}23$$

Now:

$$f_{\mathrm{cr,p}}(compression\ \&\ bending) = \frac{14{,}23 \cdot \pi^2 \cdot 200\ 000 \cdot 5^2}{12(1 - 0{,}3^2) \cdot 85^2} = 8900{,}5\ N/mm^2$$

$$f_{\mathrm{cr,p}}(compression) = \frac{4 \cdot \pi^2 \cdot 200\ 000 \cdot 5^2}{12(1 - 0{,}3^2) \cdot 85^2} = 2501{,}90 N/mm^2$$

Since one of the sides in is pure compression we select the $f_{\mathrm{cr,p}}$ of the compression case. A lower value for $f_{\mathrm{cr,p}}$ will result in a high $\bar{\lambda}_{\mathrm{p}}$. Thus $f_{\mathrm{cr,p}} = 2501{,}90 N/mm^2$.

$$\bar{\lambda}_{\mathrm{p}} = \sqrt{329{,}55/2501{,}90} = 0{,}36 \leq 0{,}68$$

$$\frac{\varepsilon_{\mathrm{csm}}}{\varepsilon_{\mathrm{y}}} = \frac{0{,}25}{\bar{\lambda}_{\mathrm{p}}^{\ 3{,}6}} = \frac{0{,}25}{0{,}36^{3{,}6}} = 9{,}89 \leq \ \min\left(15; \frac{0{,}4 \cdot 0{,}161}{0{,}0016}\right) = \min(15;\ 40{,}25)$$

Boundries are satisfied thus: $\frac{\varepsilon_{\mathrm{csm}}}{\varepsilon_{\mathrm{y}}} = 9{,}89$.

For combined loading the following formula applies:

$$M_{\mathrm{y,Ed}} \leq M_{\mathrm{R,csm,y,Rd}} = M_{\mathrm{csm,y,Rd}} \frac{(1 - n_{\mathrm{csm}})}{(1 - 0{,}5a_{\mathrm{w}})} \leq M_{\mathrm{csm,y,Rd}}$$

In which:

$$M_{\mathrm{csm,Rd}} = \frac{W_{\mathrm{pl}} f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \left[1 + \frac{E_{\mathrm{sh}}}{E} \frac{W_{\mathrm{el}}}{W_{\mathrm{pl}}} \left(\frac{\varepsilon_{\mathrm{csm}}}{\varepsilon_{\mathrm{y}}} - 1\right) - \left(1 - \frac{W_{\mathrm{el}}}{W_{\mathrm{pl}}}\right) \Big/ \left(\frac{\varepsilon_{\mathrm{csm}}}{\varepsilon_{\mathrm{y}}}\right)^{\alpha}\right]$$

For α, see table D.2.

$$M_{\mathrm{csm,Rd}} = \frac{63{,}73 \cdot 10^3 \cdot 329{,}55}{1{,}1} \left[1 + \frac{1700{,}07}{200\,000} \cdot \frac{53{,}36 \cdot 10^3}{63{,}73 \cdot 10^3} (9{,}89 - 1) - \left(1 - \frac{53{,}36 \cdot 10^3}{63{,}73 \cdot 10^3}\right) / (9{,}89)^2\right]$$

$$M_{\mathrm{csm,Rd}} = 20{,}27\ kNm$$

NOTE: We can already see that the profile can NOT resist the applied bending moment of 25 kNm. To be complete we will finalize the calculation.

RHS with $\bar{\lambda}_{\mathrm{p}} \leq 0{,}60$ subject to combined loading:

$$\left[\frac{M_{\mathrm{y,Ed}}}{M_{\mathrm{R,csm,y,Rd}}}\right]^{\alpha_{\mathrm{csm}}} \leq 1$$

In which:

$$\alpha_{\mathrm{csm}} = 1{,}66/(1 - 1{,}13 n_{\mathrm{csm}}{}^2)$$

$$M_{\mathrm{R,csm,y,Rd}} = M_{\mathrm{csm,y,Rd}} \frac{(1 - n_{\mathrm{csm}})}{(1 - 0{,}5a_{\mathrm{w}})} \leq M_{\mathrm{csm,y,Rd}}$$

Where:

$$n_{\mathrm{csm}} = \frac{N_{\mathrm{Ed}}}{N_{\mathrm{csm,Rd}}} = \frac{250}{584{,}76} = 0{,}43$$

$$N_{\mathrm{csm,Rd}} = \frac{A f_{\mathrm{csm}}}{\gamma_{\mathrm{M0}}} = \frac{1818{,}45 \cdot 353{,}73}{1{,}1} = 584{,}76\ kN$$

$$f_{\mathrm{csm}} = f_{\mathrm{y}} + E_{\mathrm{sh}} \varepsilon_{\mathrm{y}} \left(\varepsilon_{\mathrm{csm}} / \varepsilon_{\mathrm{y}} - 1\right) = 329{,}55 + 1700{,}07 \cdot 0{,}0016(9{,}89 - 1) = 353{,}73\ N/mm^2$$

$$\alpha_{\mathrm{csm}} = 1{,}66/(1 - 1{,}13 \cdot 0{,}43^2) = 2{,}10$$

$$a_{\mathrm{w}} = \frac{(h - 3t)2t}{A} = \frac{(100 - 3 \cdot 5) \cdot 2 \cdot 5}{1818{,}45} = 0{,}47$$

$$M_{\mathrm{R,csm,y,Rd}} = 20{,}27\ \cdot 10^6 \cdot \frac{(1 - 0{,}43)}{(1 - 0{,}5 \cdot 0{,}47)} = 15{,}1\ kNm \leq 20{,}27\ kNm$$

Again, we know that the profile can NOT resist the eccentric loading of 250 kN.

End of calculation.