

The Continuous Strength Method for Stainless Steel Columns

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Abstract

The Continuous Strength Method provides excellent resistance predictions of stocky cross-sections under different loading conditions as it accounts for enhanced material properties, although the method is currently limited to cross-sectional resistance. The axial stress in short columns potentially exceeds the yield strength as members are stable enough to allow partial yielding of non-slender cross-sections and the flexural buckling resistance prediction of these short columns is therefore usually underestimated, since equations currently codified in standards do not account for strain hardening effects. This paper presents the extension of the Continuous Strength Method to stainless steel Rectangular and Square Hollow Section columns, allowing strain hardening effects to be incorporated in the resistance prediction of members subjected to compression. The proposed approach is equivalent to the traditional approach for columns codified in different standards based on the Ayrton-Perry formulation but considers a different generalized imperfection parameter that depends on the cross-sectional slenderness and considered material, so strain hardening effects are directly introduced in the formulation. Results show that the proposed Continuous Strength Method approach provides improved predictions of the resistance of Rectangular and Square Hollow Section columns for all stainless steel grades. The reliability of the proposed approach has been demonstrated through the corresponding statistical analyses. Given the purely analytic nature of the proposed approach, it can be used for several materials, such as stainless steel, carbon steel and aluminium, and cross-section types for which the Continuous Strength Method has been defined. Finally, the better estimation of the flexural buckling behaviour of columns will have a direct impact on the accuracy of beam-column checks.

Keywords

buckling curve, column, Continuous Strength Method, flexural buckling, RHS and SHS, stainless steel

1 Introduction

The efficient design of structures is one of the mainstays in design practise, regardless the considered construction material. This efficiency depends on the adopted structural typology and general design, which rely on the engineers' capacity of searching for optimum solutions. However, the efficiency of a structure also depends on the design provisions codified in the different standards. Given the high material cost of stainless steel in comparison to carbon steel, the development of efficient design expressions that include all specific features of this corrosion-resistant material is crucial to incorporate it in the routine engineering practise. Improving the expressions codified in design standards (e.g. EN 1993-1-4 ^[1], AS/NZS4673 ^[2], SEI/ASCE 8-02 ^[3]) and accounting for the non linear stress-strain response and strain hardening effects, stainless steels would lead to more efficient, economic and sustainable structural designs.

The Continuous Strength Method (CSM) is a deformation-based design method that accounts for strain hardening effects in non-slender cross-sections, allowing a more efficient design of stainless steel structures. Currently, the method provides design expressions for the cross-sectional resistance against compression, bending and combined loading conditions, and design prescriptions have also been proposed for the analysis of statically indeterminate structures. However, the extension of the CSM to member behaviour is still being developed. Several research works on the behaviour of stainless steel members subjected to combined loading ^[4,5] highlighted the need of providing accurate end point resistance predictions in order to provide adequate resistance estimations. Zhao *et al* ^[5] proposed to incorporate the CSM bending moment resistance predictions in the design of stainless steel beam-columns with stocky cross-sections, but no modification was proposed for the flexural buckling resistance. First efforts to extend the CSM to stainless steel columns were published by ^[6], although the method is based on several modification factors that account for the nonlinear material response and cross-section slenderness, fitted from experimental and numerical data on stainless steel columns. Other research works accounting for strain hardening effects on stainless steel members, but based on the Direct Strength Method approach, can be also found ^[7].

This paper presents a consistent new approach for stainless steel members subjected to compression based on the CSM that allows the incorporation of strain hardening effects in the column resistance prediction for members stable enough to allow partial yielding of the cross-section. It is demonstrated that the new proposal provides more accurate predictions of the flexural buckling resistance of stainless steel columns and it has been statistically validated.

2 The Continuous Strength Method for Cross-section Resistance

The Continuous Strength Method (CSM) is a deformation-based design method that accurately predicts the cross-sectional resistance of several metallic materials such as stainless steel, carbon steel and aluminium. The method was originally developed for stocky cross-sections and allowed the implementation of strain hardening effects in the calculation of resistances. However, recent research works^[8,9] extended the CSM also for slender cross-sections, governed by local buckling effects, providing a direct design approach where the reduction of the gross-section resistance is determined by a strength curve, avoiding the calculation of effective widths, usually tedious and iterative procedures. Several research works have demonstrated that the method provides excellent cross-sectional resistance predictions for non-slender cross-sections for various loading conditions, being more accurate than the expressions currently codified in different standards^[1,2,3]. However, and according to the Fourth Edition of the Design Manual for Structural Stainless Steel^[10], for symmetrical sections the CSM shows little advantage for cross-section slenderness greater than 0.68 for plated sections such as the RHS and SHS considered in this study, although significant advantages are observed for asymmetrical sections across the full range of cross-section slenderness.

The CSM is based on a base curve that determines the maximum strain that a cross-section can reach ε_{CSM} evaluated in terms of its relative slenderness $\bar{\lambda}_p$ and the yield strain ε_y , as shown in Eq. (1a) and Eq. (1b) for stocky and slender cross-sections, respectively, adjusted considering stub column and beam test data^[8,9,11]. The $\bar{\lambda}_p = 0.68$ limit is adopted given that, beyond this limit, there is no significant benefit of considering material strain hardening effects and local buckling effects govern cross-sectional failure. The cross-sectional slenderness for each loading case can be calculated from Eq. (2), where σ_{cr1} is the critical local buckling stress, obtained from the relevant buckling mode in an eigenvalue analysis and $\sigma_{0.2}$ corresponds to the 0.2% proof stress. At the same time $\bar{\lambda}_p$ can be also calculated according to EN 1993-1-4^[1] for the most slender plate element in the cross-section. It should be noted that the former procedure accounts for element interaction whereas the latter does not. Two upper bounds, shown in Eq. (1a), are also provided on the predicted deformation capacity ε_{CSM} for stocky cross-sections. The first limit corresponds to the material ductility requirements in EN 1993-1-1^[12] while the second ensures that resistances are not over-predicted due to the adopted bilinear stress-strain material model. For austenitic and duplex stainless steel grades C=0.1 was adopted, but C=0.4 was defined for ferritics, and ε_u corresponds to the ultimate strain.

$$\frac{\varepsilon_{CSM}}{\varepsilon_y} = \begin{cases} \frac{0.25}{\bar{\lambda}_p^{3.6}} \leq \min\left(15, \frac{C \cdot \varepsilon_u}{\varepsilon_y}\right) & \bar{\lambda}_p \leq 0.68 \\ \frac{1}{\bar{\lambda}_p^{1.05}} \left(1 - \frac{0.222}{\bar{\lambda}_p^{1.05}}\right) & \bar{\lambda}_p > 0.68 \end{cases} \quad (1a)$$

$$\bar{\lambda}_p = \sqrt{\frac{\sigma_{0.2}}{\sigma_{cr1}}} \quad (3)$$

Once the maximum strain that the cross-sections can attain ε_{CSM} is calculated, the compression and bending resistance of stocky and slender cross-sections can be determined from Eq. (3) and Eq. (4), respectively. In these equations N_y is the squash load of the gross cross-section, M_{pl} and M_{el} are the plastic and elastic bending moment capacities and W_{pl} and W_{el} correspond to the plastic and elastic modulus, respectively. In addition, E is the Young's modulus and E_{sh} is the strain hardening modulus calculated from Eq. (5) for the different stainless steel grades, based on the most relevant material parameters defined before and where σ_u is the tensile strength. The simplified bilinear material model including strain hardening effects was proposed for austenitic and duplex stainless steel grades by^[11], which was later adapted for the less ductile ferritic grades^[13].

$$N_{CSM} = \begin{cases} N_y \left[1 + \frac{E_{sh}}{E} \left(\frac{\varepsilon_{CSM}}{\varepsilon_y} - 1 \right) \right] & \bar{\lambda}_p \leq 0.68 \\ N_y \left(\frac{\varepsilon_{CSM}}{\varepsilon_y} \right) & \bar{\lambda}_p > 0.68 \end{cases} \quad (3)$$

$$M_{CSM} = \begin{cases} M_{pl} \left[1 + \frac{E_{sh}}{E} \frac{W_{el}}{W_{pl}} \left(\frac{\varepsilon_{CSM}}{\varepsilon_y} - 1 \right) - \left(1 - \frac{W_{el}}{W_{pl}} \right) \left(\frac{\varepsilon_{CSM}}{\varepsilon_y} \right)^{-2} \right] & \bar{\lambda}_p \leq 0.68 \\ M_{el} \left(\frac{\varepsilon_{CSM}}{\varepsilon_y} \right) & \bar{\lambda}_p > 0.68 \end{cases} \quad (4)$$

$$E_{sh} = \frac{\sigma_u - \sigma_{0.2}}{0.16\varepsilon_u - \varepsilon_y} \quad \text{for austenitic and duplex grades} \quad (5a)$$

$$= \begin{cases} \frac{\sigma_u - \sigma_{0.2}}{0.45\varepsilon_u - \varepsilon_y} E_{sh} & \text{for } \frac{\varepsilon_y}{\varepsilon_u} \leq 0.45 \\ 0 & \text{for } \frac{\varepsilon_y}{\varepsilon_u} > 0.45 \end{cases} \quad \text{for ferritic grades} \quad (5b)$$

Several research works ^[14,15] also extended the CSM approach for stocky cross-sections to more general loading conditions such as combined axial compression and bending moment. Proposals are based on the more accurate compression and bending end points calculated according to the CSM with different approaches based on the interaction equations given in EN 1993-1-1 ^[12] and all studies demonstrated that they provide excellent strength predictions.

The CSM provides excellent resistance prediction of the cross-sectional resistance of a number of cross-section types and materials, both for stocky and slender sections. In this process of continuous development, a new CSM approach is proposed to provide a consistent and accurate method applicable not only for cross-sections, but also for structural members.

3 Gathered Experimental Data and FE Ultimate Strengths

The analysis presented in this paper has been based on an extensive experimental and numerical database of stainless steel RHS and SHS members subjected to compression. This section presents the gathered experimental data collection and summarises the most relevant aspects of the conducted Finite Element (FE) numerical analysis.

3.1 Experimental data collection

The behaviour of stainless steel RHS and SHS members subjected to compression has been extensively analysed through different experimental programmes during last decades, including several stainless steel grades. Column tests on austenitic ^[16-21], ferritic ^[4,22,23] and duplex (or lean duplex) ^[24-27] stainless steels have been considered in the analysis.

3.2 FE model validation

The column FE models utilised in the parametric study were performed by the general purpose software Abaqus and they were first validated against the experimental results reported in Arrayago *et al.* ^[4]. The mid-surfaces of the cross-sections were modelled by using four-node shell elements with reduced integration S4R, widely used when modelling cold-formed stainless steel cross-sections. The computational efficiency and reliability of the results was guaranteed by a mesh convergence study and the analyses were conducted with 5 mm long shell elements. The initial geometric imperfections with the shape of elastic buckling modes were introduced into the model by conducting a previous linear elastic buckling analysis, and the introduced imperfection amplitudes were those measured in the specimens and reported in [4]. The non-linear problem was solved by using a modified Riks analysis.

In order to model the pin-ended boundary conditions of the tests, the edge elements at the ends of the members were kinematically coupled and connected to two reference points. These reference points were defined according to what was described in [4], assuming the effective length of the members equal to the distance between knife-edges, 50 mm away from each specimen end. All degrees of freedom except the rotation around minor axis were restrained at the lower reference point, while longitudinal displacement and minor axis rotations were set free in the upper one. The load was then introduced as an imposed displacement at the upper reference point and no restrictions were defined in the rest of the nodes.

The validation of the FE models was performed considering the flat and corner regions of the cross-sections, assigning the corresponding measured stress-strain properties to each region. Corner regions were also extended to the adjacent flat parts by a length equal to two times the thickness of the element, as assumed in [24,25]. According to [21] coupons curve longitudinally when cut from the tube and return to their original straight shape when they are gripped in the testing machine. During the straightening process, the bending residual stresses are approximately reintroduced, so the obtained stress-strain curves include the bending residual stresses and do not need to be reintroduced into the FE models. The material parameters describing the behaviour of flat and corner parts can be found in [4].

Experimental load-lateral deflection curves have been compared to the corresponding FE results, which is presented in Figure 1 for the considered columns and in Table 1, where the mean values and the coefficients of variation (COV) of the numerical-to-experimental ratios of the ultimate loads and the corresponding lateral deflections are reported. These comparisons demonstrate that the derived numerical analyses are capable of accurately predicting the ultimate loads and replicating the full experimental histories. It is also remarkable that the failure modes of the obtained FE models are in good agreement with experimental results.

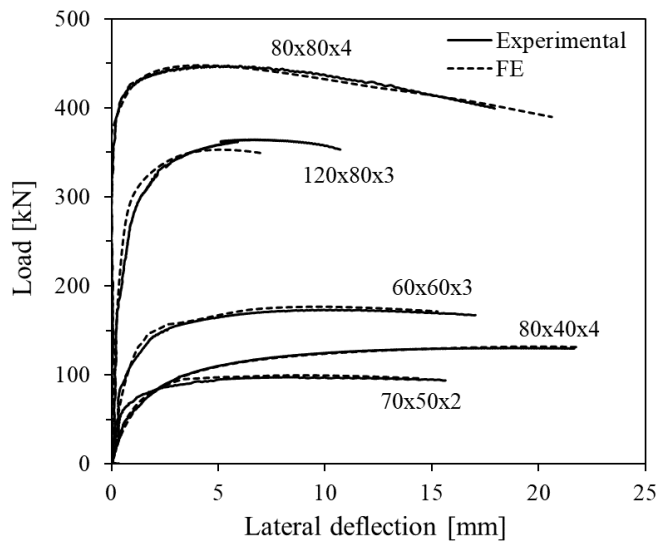


Fig. 1 Comparison of experimental and FE curves for ferritic stainless steel columns

Table 1 Comparison between experimental and FE results for ferritic stainless steel columns

Specimen	$F_{u,FE}/F_{u,exp}$	$d_{u,FE}/d_{u,exp}$
80x80x4	1.00	0.93
60x60x3	1.02	0.99
80x40x4	1.01	1.03
120x80x3	0.97	0.80
70x50x2	1.02	1.00
Mean	1.00	0.95
COV	0.021	0.096

3.3 Numerical parametric studies

Once the numerical model was validated against experimental results for stainless steel RHS and SHS members subjected to compression, a parametric study was developed through a combination of Python and Abaqus tools.

Initial global imperfection amplitudes have great influence on the flexural buckling behaviour of stainless steel columns and the values considered in the parametric study need to be carefully defined, although different values have been considered in the numerical studies conducted during this last decade. Considering that the measured initial imperfection amplitudes of the members tested in the experimental programme reported in [4] ranged between $L/1000$ and $L/2000$ and that residual stresses are implicitly incorporated in the models through the measured stress-strain material definition according to [21], the $L/1500$ magnitude has been considered as imperfection amplitude for the parametric study. Local initial imperfections were also included with amplitudes predicted from the modified Dawson and Walker model [28] proposed by Gardner and Nethercot [29].

The parametric study focus primarily on ferritic stainless steel, although results are also presented for austenitic and duplex grades. Material properties were obtained from the results reported in [4] for ferritics, while those used in the parametric study by [5] were considered for austenitic and duplex stainless steels. The most relevant material parameters are presented in Table 2 where E is the Young's modulus, $\sigma_{0.2}$ is the proof stress corresponding to 0.2% plastic strain, σ_u is the tensile strength and ϵ_u is the corresponding ultimate strain. Strain hardening exponents n and m are also provided. The stress-strain curves used for the numerical analyses were obtained using the stress-strain curve formulation presented in Arrayago *et al.* [30] in combination with the parameters shown in Table 2.

Table 2 Material parameter definition for parametric studies

Stainless steel	E [GPa]	$\sigma_{0.2}$ [MPa]	σ_u [MPa]	ϵ_u [%]	n	m
Austenitic	197.8	417	651	35.9	5.5	3.7
Ferritic	185.7	490	533	4.8	11.0	3.2
Duplex	201.3	707	874	19.1	5.6	4.9

The parametric study on stainless steel members comprised a variety of different RHS and SHS, whose minor axis flexural buckling behaviour was investigated, and where all models were defined according to the general assumptions described in the previous section. Stocky RHS and SHS cross-sections with aspect ratios h/b equal to 1, 1.2, 1.5 and 2 were considered, with thicknesses ranging between 2 mm and 4 mm and modelling different slendernesses, ranging from 0.25-0.75, for each cross-section.

4 The Continuous Strength Method for Members in Compression: New Approach

The Continuous Strength Method (CSM) provides excellent resistance predictions of stocky cross-sections, as it accounts for strain hardening effects and provides a direct and easy procedure to account for local buckling effects in slender cross-sections, avoiding the calculation of effective cross-section properties. However, the method is limited to cross-sectional resistance predictions in its current definition.

The failure of slender stainless steel columns occurs at low stresses since the applied load reaches the elastic critical force for flexural buckling before the axial stress exceeds the 0.2% proof stress, or even below the proportional limit of the material. However, for relatively short columns, the axial stress will potentially exceed the yield strength as the member is stable enough to allow partial yielding of the cross-section, especially for members with stocky cross-sections. The resistance prediction of these short columns is therefore usually underestimated since equations currently codified in standards do not account for strain hardening effects, and therefore, a new CSM-based approach is proposed for stainless steel RHS and SHS columns.

4.1 Flexural buckling in standards

Traditional design methods for column behaviour provided in different standards, such as EN 1993-1-4 ^[1] and AS/NZS4673 ^[2], are based on the Ayrton-Perry buckling formulation established in EN 1993-1-1 ^[12] for carbon steel members, where the column strength is determined by reducing the cross-section squash load due to flexural buckling effects, as in Eq. (6). γ_{M1} is the partial factor for resistance of members to instability and $\bar{\lambda} = \sqrt{N_{c,Rk}/N_{cr}}$ is the member slenderness, calculated from the cross-sectional compression resistance $N_{c,Rk}$ and the elastic flexural buckling load of the column N_{cr} .

The reduction factor χ is calculated from the corresponding buckling curve as in Eqs. (7) and (8) and depends on the considered standard or alternative approach. The buckling curve codified in EN 1993-1-4 ^[1] corresponds to the buckling curve c for all stainless steel hollow sections, with $\eta = \alpha(\bar{\lambda} - \bar{\lambda}_0)$, $\alpha = 0.49$ and $\bar{\lambda}_0 = 0.4$, while AS/NZS4673 ^[2] is based on a nonlinear generalised imperfection factor $\eta = \alpha[(\bar{\lambda} - \bar{\lambda}_1)^\beta - \bar{\lambda}_0]$ and provides different buckling curves for several stainless steel grades through the α , β , $\bar{\lambda}_0$ and $\bar{\lambda}_1$ parameters.

$$N_{b,Rd} = \frac{\chi N_{c,Rk}}{\gamma_{M1}} \quad (6)$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1 \quad (7)$$

$$\phi = 0.5 \cdot [1 + \eta + \bar{\lambda}^2] \quad (8)$$

AS/NZS4673 ^[2] also allows the use of the tangent modulus approach in the prediction of the flexural buckling resistance, where the nonlinear stress-strain response of the material is considered by allowing a gradual yielding through the use of the tangent modulus E_t corresponding to the buckling stress. However, since the calculation of this tangent modulus involves an iterative design procedure and the method does not consider member imperfections, the tangent modulus approach has not been analysed in this paper.

Recent reliability analyses by Afshan *et al.* [31] on the flexural buckling curve codified in EN 1993-1-4 [1] for stainless steel RHS and SHS columns demonstrated that the partial safety factors γ_{M1} derived exceeded the recommended value of 1.1, indicating that lower buckling curves might be required. It was also found out that different buckling curves were necessary for the different stainless steel grades. These revised curves are similar to buckling curve *c*, since they adopt the same imperfection factor $\alpha = 0.49$ for all stainless steels, but consider different limiting slendernesses, $\bar{\lambda}_0 = 0.3$ for austenitic and duplex stainless steel alloys and $\bar{\lambda}_0 = 0.2$ for ferritics. The revised buckling curves have already been included in the Fourth Edition of the Design Manual for Structural Stainless Steel [10].

4.2 Development of the Continuous Strength Method approach for columns

The new CSM-based approach proposed in this paper for columns has been developed using an analytic approach in order to obtain a consistent and complete proposal extendable to any cross-section or material. For this, a pin-ended column is considered, with a half sine-wave initial geometrical imperfection with magnitude e_0 , subjected to an axial compression load. From the classical elastic flexural equilibrium equation accounting for initial geometrical imperfections, a second order in-plane elastic check of the most heavily loaded cross-section on the member (at mid-span section) becomes:

$$\frac{N_{b,Rd}}{N_{c,Rd}} + \frac{1}{\left(1 - \frac{N_{b,Rd}}{N_{cr}}\right)} \frac{N_{b,Rd} \cdot e_0}{M_{c,Rd}} = 1 \quad (9)$$

where $N_{c,Rd}$ and $M_{c,Rd}$ are the cross-sectional compression and bending moment resistances, N_{cr} is the elastic critical buckling load of the member and $N_{b,Rd}$ is the axial compressive load that causes the failure of the column, which can be considered the flexural buckling resistance of the member. For simplicity, all safety factors will be considered equal to one in the development of the new approach and the different assessments presented in the following sections. According to EN 1993-1-4 [1], $N_{c,Rd}$ and $M_{c,Rd}$ are limited to the yield strength regardless of the slenderness of cross-sections, without allowing any strain hardening considerations. On this basis, the compression and bending moment resistances according to the CSM given in Eqs. (3) and (4) can be adopted in Eq. (9) to allow enhanced material properties to be included in the formulation, as in Eq. (10), where $N_{b,CSM}$ is the flexural buckling resistance of column according to the CSM approach.

$$\frac{N_{b,CSM}}{N_{CSM}} + \frac{1}{\left(1 - \frac{N_{b,CSM}}{N_{cr}}\right)} \frac{N_{b,CSM} \cdot e_0}{M_{CSM}} = 1 \quad (10)$$

Considering that $\sigma_{b,CSM} = N_{b,CSM}/A$, $\sigma_{CSM} = N_{CSM}/A$ and $\sigma_{cr} = N_{cr}/A$, Eq. (10) can be re-written in terms of stresses, as for Eq. (11), adopting the auxiliary function f^* defined in Eq. (12), where f corresponds to the ratio between the CSM and plastic bending moment capacities, calculated from Eq. (4), $f = M_{CSM}/M_{pl}$. Note that σ_{CSM} corresponds to the cross-sectional compression strength while $\sigma_{b,CSM}$ stands for the flexural buckling failure stress.

$$\frac{\sigma_{b,CSM}}{\sigma_{CSM}} + \frac{1}{\left(1 - \frac{\sigma_{b,CSM}}{\sigma_{cr}}\right)} \frac{\sigma_{b,CSM} \cdot e_0}{\frac{W_{pl}}{A} \sigma_{CSM} \cdot f^*} = 1 \quad (11)$$

$$f^* = f \frac{\sigma_{0.2}}{\sigma_{CSM}} \quad (12)$$

Operating on Eq. (11) and assuming a new auxiliary function $g = f^* \psi$, where $\psi = W_{pl}/W_{el}$ is the shape factor of the cross-section, Eq. (13) can be derived. This expression is the same Ayrton-Perry considered in the traditional flexural buckling approach, but with a different generalized imperfection parameter η^* . Introducing the slenderness plateau $\bar{\lambda}_0$ and adopting the non-dimensional member slenderness $\bar{\lambda} = \lambda/\lambda_E$ and the following relationships $\gamma = L/e_0$, $\lambda = L/i$, $i^2 = I/A$, $W_{el} = I/v$, the modified generalized imperfection factor can be expressed as in Eq. (14), where η is the generalized imperfection factor considered in the Ayrton-Perry formulation adopted in the Eurocode. In these expressions λ_E is the Euler's slenderness, L is the length of the column, I is the relevant second moment of area, A is the gross-section area and v is the distance from neutral axis to extreme fibres.

$$(\sigma_{CSM} - \sigma_{b,CSM}) \cdot (\sigma_{cr} - \sigma_{b,CSM}) = \sigma_{b,CSM} \cdot \sigma_{cr} \cdot \eta^* \text{ with } \eta^* = \frac{e_0 \cdot A}{W_{el} \cdot g} \quad (13)$$

$$\eta^* = \frac{\alpha}{g} (\bar{\lambda} - \bar{\lambda}_0) = \frac{1}{g} \alpha (\bar{\lambda} - \bar{\lambda}_0) = \frac{1}{g} \eta \text{ with } \alpha = \frac{\pi \sqrt{E/\sigma_{0.2}}}{\gamma \cdot \frac{i}{v}} \quad (14)$$

From Eq. (13), and considering $\chi_{CSM} = \frac{\sigma_{b,CSM}}{\sigma_{CSM}}$, $\sigma_{cr} = \frac{\sigma_{CSM}}{\bar{\lambda}^2}$, $\frac{\sigma_{b,CSM}}{\sigma_{cr}} = \chi_{CSM} \cdot \bar{\lambda}^2$, Eq. (15) can be derived, whose smallest root corresponds to the reduction factor χ_{CSM} that provides the CSM flexural buckling resistance using Eqs. (16)-(19). Finally, the CSM flexural buckling resistance $N_{b,CSM}$ can be calculated from Eq. (19).

$$\chi_{CSM}^2 \cdot \bar{\lambda}^2 - \chi_{CSM} [1 + \eta^* + \bar{\lambda}^2] + 1 = 0 \quad (15)$$

$$\chi_{CSM} = \frac{1}{\phi_{CSM} + \sqrt{\phi_{CSM}^2 - \bar{\lambda}^2}} \quad (16)$$

$$\phi_{CSM} = 0.5 \cdot [1 + \eta^* + \bar{\lambda}^2] \quad (17)$$

$$\eta^* = \frac{1}{g} \alpha (\bar{\lambda} - \bar{\lambda}_0) = \frac{\sigma_{CSM} W_{el}}{M_{CSM}} \alpha (\bar{\lambda} - \bar{\lambda}_0) \quad (18)$$

$$N_{b,CSM} = \frac{\chi_{CSM} \cdot N_{CSM}}{\gamma_{M1}} \quad (19)$$

The proposed CSM approach for flexural buckling resistance defined through Eqs. (16)-(19) is equivalent to the traditional approach for columns codified in different standards ^[1-3,12] described in Section 4.1, considering a different generalized imperfection parameter η^* , but still based on the flexural buckling curve concept through parameters α and $\bar{\lambda}_0$. This imperfection parameter depends on the cross-sectional slenderness and considered material, since the CSM strength σ_{CSM} and bending moment capacity M_{CSM} are needed in its definition. Hence, for columns with stocky cross-sections strain hardening effects are directly introduced in the formulation. According to the proposed formulation, stocky cross-sections with materials showing high strain hardening provide lower generalised imperfection values and therefore, higher predicted column resistances.

On the other hand, the prediction of the flexural buckling resistance of columns with slender cross-sections (showing $\bar{\lambda}_p > 0.68$) need to account for resistance reductions due to local buckling instead of strain hardening effects. As for cross-sections, this reduction can be considered through the traditional effective width method calculating the effective cross-section area, but also from the CSM strength curve defined in Eq. (1b). This second procedure is more direct, simpler and based on gross-sectional properties. The proposed CSM approach can be easily extended to columns showing slender cross-sections by adopting the auxiliary function $g = 1$, which is equivalent to considering the procedure codified in EN 1993-1-4 ^[1] with fully effective cross-section properties. The local buckling effects can be then introduced through the strength curve as in Eq. (20), where $N_{b,0}$ is the flexural buckling resistance of the fully effective cross-section. In this expression, the cross-section slenderness might be calculated from Eq. (2), although a more adequate cross-section slenderness $\bar{\lambda}_p$ definition needs to be adopted in order to obtain a better prediction of the column resistance. This new definition, given in Eq. (21), is equivalent to the λ_1 slenderness defined in the AISI-S100-16 Specification ^[32] for flexural-local buckling interaction according to the Direct Strength Method approach, which guarantees a good estimation of the susceptibility of cross-sections to local buckling. In its definition, the flexural buckling resistance of the fully effective cross-section $N_{b,0}$ is considered instead of the cross-section squash load, as well as the elastic critical local buckling load under compression $N_{cr,l}$.

$$\frac{N_{b,CSM}}{N_{b,0}} = \frac{1}{\bar{\lambda}_p^{1.05}} \left(1 - \frac{0.222}{\bar{\lambda}_p^{1.05}} \right) \quad (20)$$

$$\bar{\lambda}_p = \sqrt{\frac{N_{b,0}}{N_{cr,l}}} \quad (21)$$

Figures 2-4 present the equivalent buckling curves obtained with the new CSM approach for austenitic, ferritic and duplex stainless steel alloys, respectively, for four different cross-section slenderness values. Curves corresponding to RHS and SHS with low, medium and high cross-section slenderness values are presented. Stocky and slender sections are included, together with the flexural buckling curve c codified in EN 1993-1-4 ^[1] for stainless steel hollow sections, for comparison. It is evident from Figures 2-4 that for a given material the highest flexural buckling curves are obtained for those cross-sections showing the lowest cross-section slenderness $\bar{\lambda}_p$, obtaining higher predicted column resistances. In addition, it can be observed that buckling curves are higher for those materials for which strain hardening effects are more evident, obtaining higher flexural buckling curves for austenitic stainless steel alloys than for duplex and ferritics.

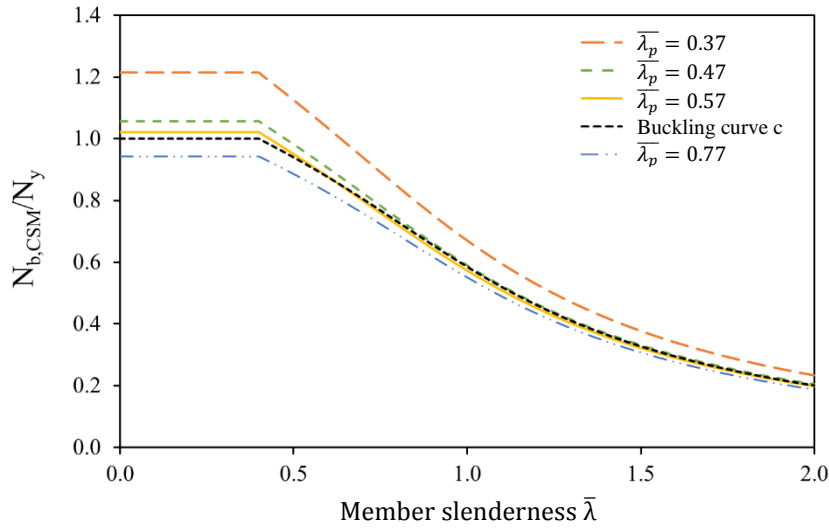


Fig. 2 Buckling curves corresponding to RHS with different slenderness for austenitic stainless steel according to the new CSM approach

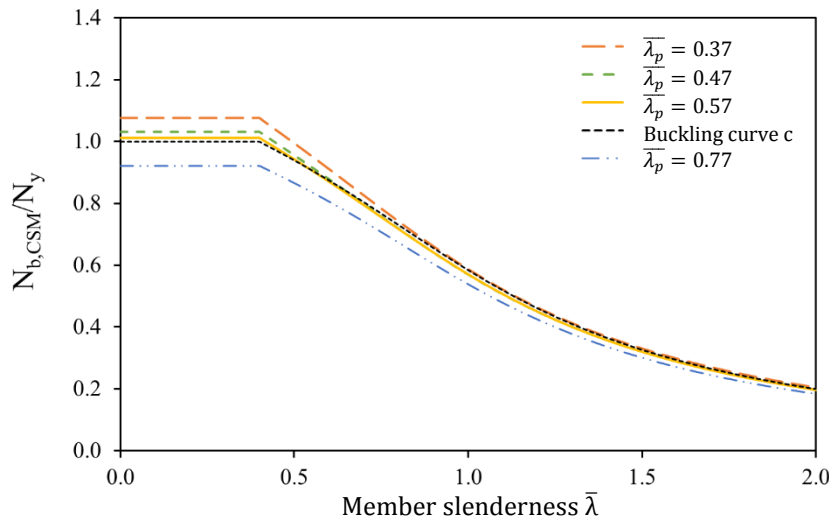


Fig. 3 Buckling curves corresponding to RHS with different slenderness for ferritic stainless steel according to the new CSM approach

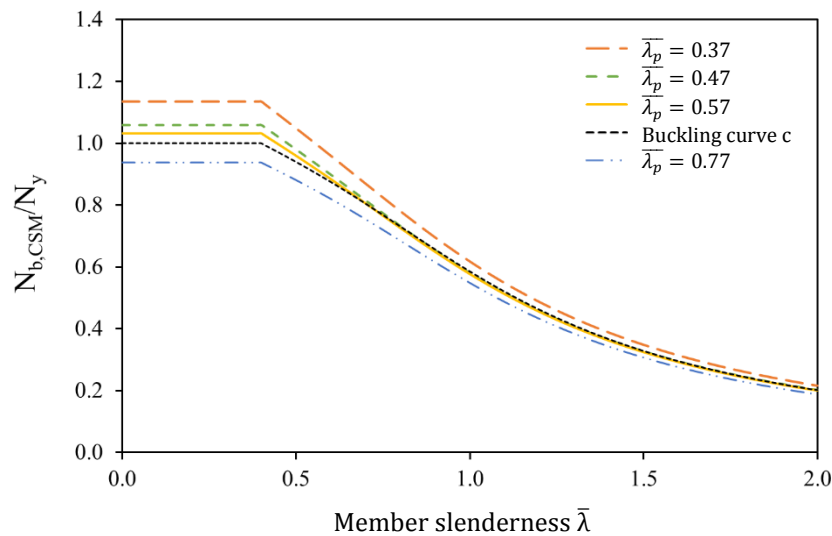


Fig. 4 Buckling curves corresponding to RHS with different slenderness for duplex stainless steel according to the new CSM approach

The purely analytic nature of the proposed approach offers a number of advantages. First, the currently codified design procedure based on flexural buckling curves is followed, without the need of introducing new methodologies. The second and most relevant advantage is that the approach can be used for all materials and cross-section types for which the CSM has been defined, including stainless steel, carbon steel and aluminium Square and Rectangular Hollow Sections, I-sections, Circular Hollow Sections. Moreover, as the definition of the most adequate flexural buckling curve changes in view of new research for each cross-section and materials, the approach will be always applicable. Finally, the better estimation of the flexural buckling behaviour of columns (together with the correct bending moment capacity prediction) will have a direct impact on the accuracy of beam-column design, since the estimation of the end points is key in the assessment of these members together with the correct interaction equation.

5 Assessment of the CSM Approach for Columns with Stocky Cross-section

The assessment of the proposed new CSM approach for stainless steel columns is presented in this section by comparing the predicted flexural buckling resistances with experimental and numerical strengths presented in section 3. Note that although the proposed approach is applicable to a number of materials and cross-section shapes for which the CSM has been developed, this paper only covers the assessment of RHS and SHS sections, which are the most commonly used stainless steel cross-sections. The assessment of the new CSM method is presented separately for columns with stocky and slender cross-sections and specimens showing cross-section slenderness lower than the CSM limit $\bar{\lambda}_p \leq 0.68$ for strain hardening considerations are considered in this section. Results are also compared to resistance predictions obtained from the codified expressions that do not consider strain hardening effects to evaluate the improvement of the new formulation.

Figure 5 shows the variation of the auxiliary function $g = M_{CSM}/(\sigma_{CSM} W_{el})$ against the cross-section slenderness $\bar{\lambda}_p$ for the different cross-sections and materials considered in this study. Note that since the generalized imperfection factor η^* decreases with increasing values of g , lower generalised imperfection values are obtained for stocky cross-sections with low $\bar{\lambda}_p$ slenderness values. In the limit, for $\bar{\lambda}_p = 0.68$, function g equals the unity and the modified buckling curve corresponds to the traditional curve.

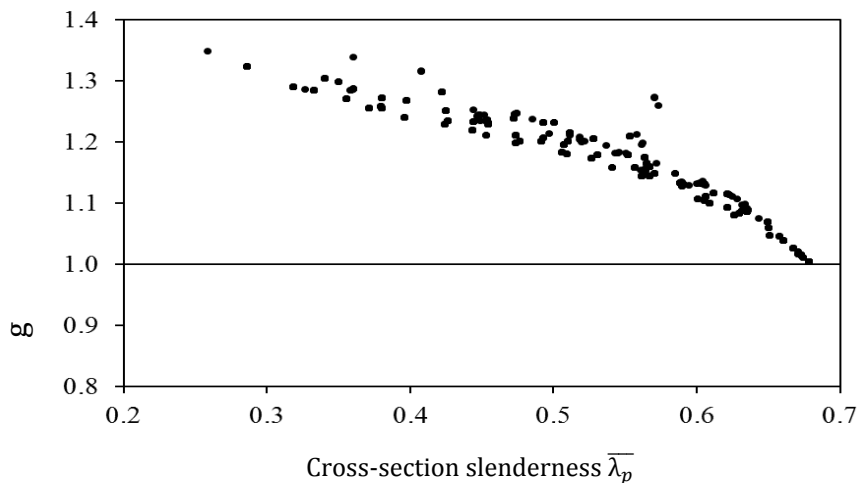


Fig. 5 Auxiliary function g , correction factor for the modified generalised imperfection factor

The accuracy of the CSM for columns has been assessed for several flexural buckling curves since the proposed methodology can be applied to different buckling curves through parameters α and $\bar{\lambda}_0$. In addition to the buckling curve c currently codified in EN 1993-1-4^[1], the revised buckling curves for stainless steel RHS and SHS proposed by Afshan *et al.*^[31] have also been analysed.

Figures 6 and 7 present the assessment of the CSM approach for columns with stocky cross-sections. In these figures, predicted-to-experimental (or FE) ratios are plotted against the corresponding member slenderness (a-figures) or cross-sectional slenderness (b-figures). The flexural buckling curve codified in EN 1993-1-4^[1] is considered in Figure 6 and the revised curves in Figure 7, for austenitic, ferritic and duplex stainless steel columns. Cross-section slenderness values have been calculated from Eq. (2), in which the elastic critical local buckling stress σ_{cr} has been obtained from CUFSM (Schafer and Adány^[33]) calculations for pure compression. From these figures, it can be seen that the prediction of column resistances is improved when the proposed CSM method is considered regardless the considered buckling curve.

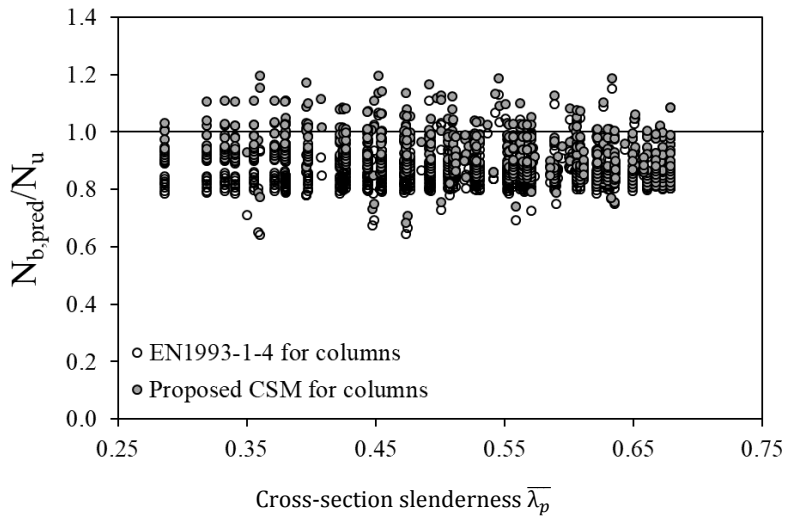
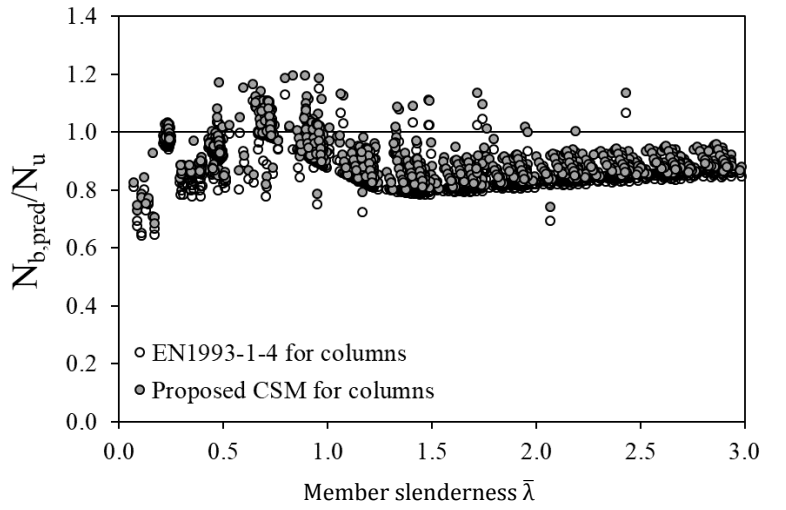
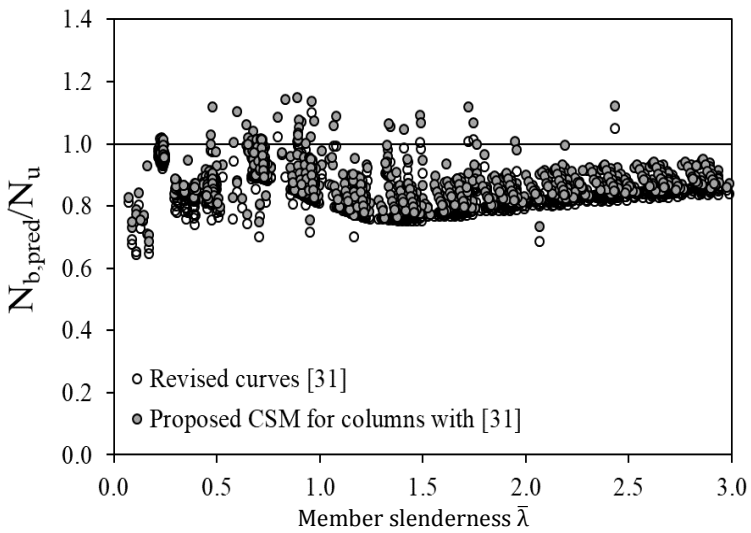


Fig. 6 Assessment of the proposed CSM approach for columns with stocky cross-section considering the flexural buckling curve codified in EN 1993-1-4^[1]



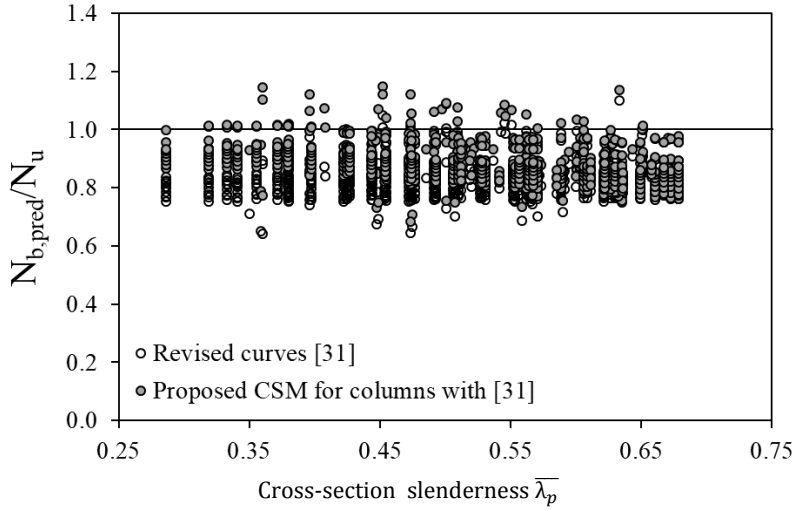


Fig. 7 Assessment of the proposed CSM approach for columns with stocky cross-section considering the revised flexural buckling curves proposed in Afshan *et al.* [31]

Results derived from the expressions that do not account for strain hardening effects ^[1,31] are compared with those obtained for the proposed CSM approach for the different buckling curves studied in order to evaluate the improvement introduced by the new method and to identify the most appropriate design approach. Table 3 presents the mean and COVs of the predicted-to-experimental (or FE) ratios for the considered flexural buckling approaches, with and without including strain hardening effects. $N_{b,EN}/N_u$ refers to the buckling curve codified in ^[1], while for the revised buckling curves proposed by ^[31] $N_{b,rev}/N_u$ is used. Results corresponding to the CSM approach and accounting for strain hardening effects based on the previous flexural buckling curves are also presented, $N_{b,CSM,EN}/N_u$ and $N_{b,CSM,rev}/N_u$.

Table 3 Assessment of design approaches for members in compression with stocky cross-sections

Grade		CSM approach		No strain hardening	
		$N_{b,CSM,EN}/N_u$	$N_{b,CSM,rev}/N_u$	$N_{b,EN}/N_u$	$N_{b,rev}/N_u$
Austenitic	Mean	0.94	0.91	0.88	0.85
	COV	0.123	0.114	0.113	0.104
Ferritic	Mean	0.92	0.88	0.87	0.83
	COV	0.076	0.067	0.075	0.066
Duplex	Mean	0.91	0.88	0.87	0.84
	COV	0.066	0.065	0.064	0.062
All	Mean	0.92	0.89	0.87	0.83
	COV	0.083	0.075	0.080	0.072

Table 3 demonstrates that the proposed CSM approach provides improved predictions of the resistance of stainless steel RHS and SHS columns for all considered buckling curves since higher $N_{b,pred}/N_u$ ratios, with similar scatter, are obtained. The consideration of the enhanced material properties in the prediction of the ultimate capacity of stainless steel columns with stocky cross-sections improves the obtained results for all stainless steel grades, although more relevant improvements are obtained for austenitic specimens than for duplex and ferritics. In accordance to Table 3, best results are observed for the buckling curve codified in EN 1993-1-4 ^[1] when strain hardening effects are included, although the corresponding reliability analyses presented in Section 7 should also be considered before any conclusion and recommendation are derived.

6 Assessment of the CSM Approach for Columns with Slender Cross-section

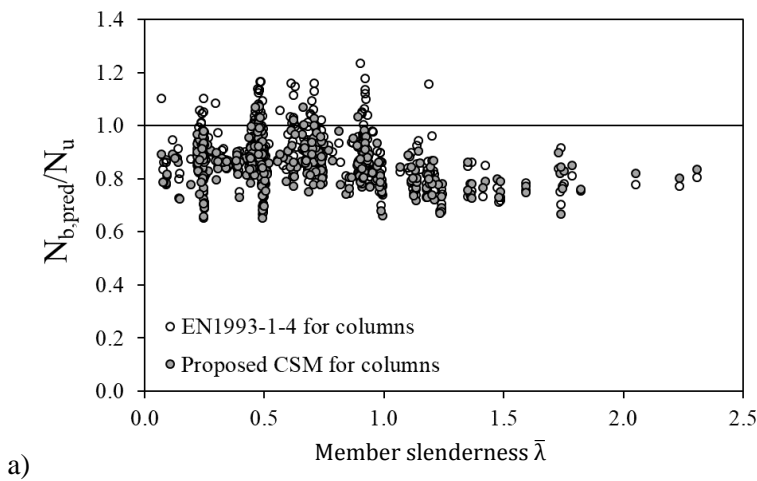
According to the review presented in Section 2, recent research works extended the CSM also to slender cross-sections, where local buckling effects in cross-sectional resistance are directly considered through a strength curve, avoiding the calculation of effective widths. This section presents the assessment of this approach when applied to stainless steel columns showing cross-section slenderness higher than the CSM limit $\bar{\lambda}_p \geq 0.68$. In the assessment, cross-section slenderness values have been calculated from Eq. (21), in which the elastic critical local buckling load $N_{cr,l}$ has also been obtained from CUFSM calculations. As for previous section, the buckling curve currently codified in EN 1993-1-4 ^[1] and the revised curves for stainless steel RHS and SHS proposed by Afshan *et al.* ^[31] have been analysed.

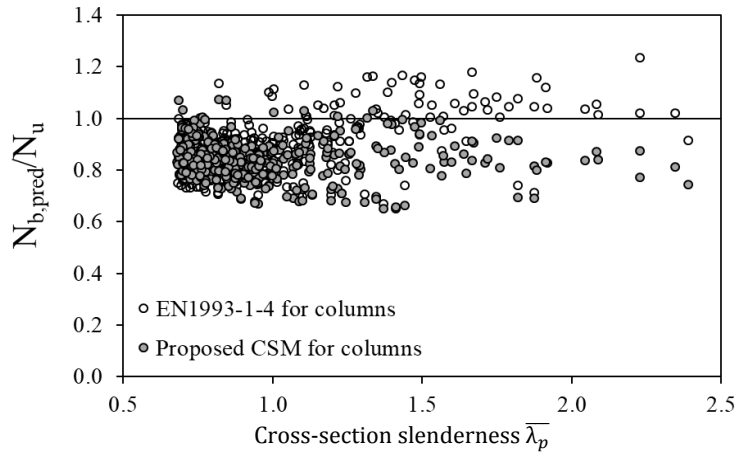
Results calculated using the CSM approach have been compared to the resistance predictions obtained from the codified expressions by means of the effective width method with the revised cross-sectional limits currently given in EN 1993-1-4 ^[1], which were proposed by Gardner and Theofanous ^[34]. Table 4 presents the mean and COVs of the predicted-to-experimental (or FE) ratios for the considered flexural buckling curves ^[1,31]. It is evident from these results that the CSM approach for stainless steel columns with slender RHS and SHS sections provides similar results and lower scatter to those obtained using the codified effective width method, as mentioned in Section 2. However, the CSM is simpler and easier to use since no effective width calculations are required, maintaining a similar level accuracy.

Table 4 Assessment of design approaches for members in compression with slender cross-sections

Grade		CSM approach		Effective width method	
		$N_{b,CSM,EN}/N_u$	$N_{b,CSM,rev}/N_u$	$N_{b,EN}/N_u$	$N_{b,rev}/N_u$
Austenitic	Mean	0.88	0.86	0.94	0.91
	COV	0.094	0.090	0.114	0.107
Ferritic	Mean	0.84	0.80	0.87	0.81
	COV	0.092	0.093	0.114	0.125
Duplex	Mean	0.84	0.82	0.90	0.87
	COV	0.069	0.071	0.114	0.115
All	Mean	0.85	0.81	0.89	0.84
	COV	0.090	0.093	0.118	0.127

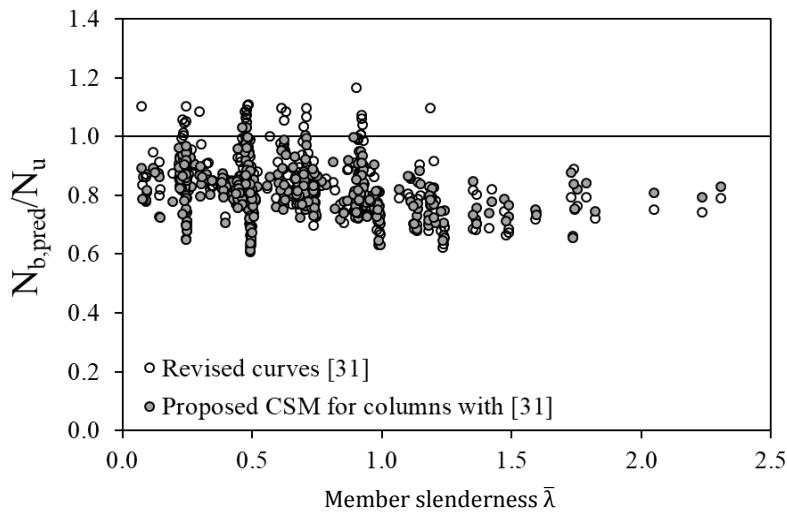
Similar conclusions can be drawn from Figures 8 and 9, where the results corresponding to the assessment of the CSM approach for stainless steel slender RHS and SHS sections are presented. In these figures, predicted-to-experimental (or FE) ratios are plotted against the corresponding member slenderness (a-figures) and against cross-sectional slenderness (b-figures), calculated from Eq. (21). Member slendernesses relative to CSM provisions have been adopted in a-figures, calculated from the gross-section properties of the cross-sections, although results corresponding to EN 1993-1-4 ^[1] have been predicted through the effective member slendernesses. Figure 8 reports results corresponding to the flexural buckling curve codified in EN 1993-1-4 ^[1], while Figure 9 considers the revised curves by [31]. These figures show the existence of some unsafe predictions corresponding to both buckling curves, explaining the higher predicted-to-experimental (or FE) ratios and scatter for the effective width approach, being this fact more evident for the buckling curve codified in EN 1993-1-4 ^[1].



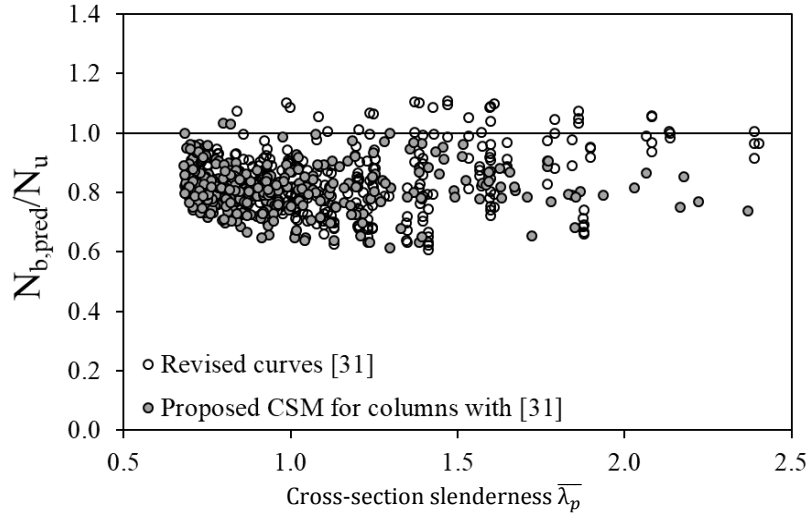


b)

Fig. 8 Assessment of the proposed CSM approach for columns with slender cross-section considering the flexural buckling curve codified in EN 1993-1-4 ^[1]



a)



b)

Fig. 9 Assessment of the proposed CSM approach for columns with slender cross-section considering the revised flexural buckling curves proposed in Afshan *et al.* [31]

7 Reliability Analysis

The reliability of the proposed CSM approach for columns is assessed through the corresponding statistical analyses for stainless steel RHS and SHS columns for the considered flexural buckling curves ^[1,31]. The statistical calibration of the proposed approach has been derived according to EN 1990, Annex D ^[35] specifications, where the statistical parameters corresponding to the material and geometrical variations of the different stainless steel grades have been extracted from Afshan *et al.* ^[36]. The considered material overstrength ratios are 1.3 for austenitic stainless steel, 1.2 for ferritics and 1.1

for duplex and lean duplex grades, with COVs equal to 0.060, 0.045 and 0.030 respectively, and the COV of the geometric properties was taken as 0.050.

The statistical analyses of the new approach for stainless steel RHS and SHS columns were derived according to EN 1990, Annex D ^[35] following the steps described in Tankova *et al.* ^[37] and the summary of the most relevant parameter values are presented in Table 5 and Table 6 for stocky and slender cross-sections, respectively. In this Table, b is the mean value of the correction factor, V_δ is the coefficient of variation of the errors of each approach relative to the experimental results and V_r is the combined coefficient of variation.

Table 5 Summary of the reliability analysis results for the CSM approach for members in compression with stocky cross-section

	Grade	b	V_δ	V_r	γ_{M1}
EN 1993-1-4 ^[1] buckling curve	Austenitic	1.113	0.122	0.145	1.08
	Ferritic	1.035	0.074	0.100	1.10
	Duplex	1.126	0.066	0.089	1.03
Revised buckling curves ^[31]	Austenitic	1.143	0.113	0.138	1.03
	Ferritic	1.101	0.066	0.094	1.01
	Duplex	1.178	0.066	0.088	0.99

Table 6 Summary of the reliability analysis results for the CSM approach for members in compression with slender cross-section

	Grade	b	V_δ	V_r	γ_{M1}
EN 1993-1-4 ^[1] buckling curve	Austenitic	1.102	0.097	0.125	1.08
	Ferritic	1.205	0.091	0.113	1.02
	Duplex	1.194	0.069	0.090	0.99
Revised buckling curves ^[31]	Austenitic	1.136	0.092	0.121	1.03
	Ferritic	1.280	0.089	0.112	0.95
	Duplex	1.236	0.073	0.093	0.97

According to the results gathered in Table 5 and Table 6, the proposed CSM approach for stainless steel columns subjected to compression can be safely applied for stocky and slender cross-sections, and for the different grades considered, since the calculated γ_{M1} values lay below the partial safety factor γ_{M1} currently codified in EN 1993-1-4 ^[1], equal to 1.10.

8 Conclusions

This paper presents a new design approach for the flexural buckling resistance of stainless steel columns based on the Continuous Strength Method (CSM). Limited to cross-sectional behaviour, the CSM provides excellent resistance predictions for stocky cross-sections as strain hardening effects are included, as well as for slender cross-sections, where effective width calculations are avoided. The new approach presented in this paper extends the CSM to member behaviour and allows considering strain hardening and local buckling effects for stainless steel RHS and SHS columns. The proposed approach is equivalent to the traditional approach for columns codified in standards based on the Ayrton-Perry formulation but considers a different generalised imperfection parameter that depends on the cross-sectional slenderness and considered material, introducing strain hardening effects directly in the formulation. Since the proposed procedure is based on a certain buckling curve, the flexural buckling curve codified in EN 1993-1-4 ^[1] and the revised curves proposed by Afshan *et al.* ^[31] have been analysed in this paper. The assessment of experimental and numerical results showed that the proposed CSM approach provides improved predictions of the resistance of RHS and SHS columns with stocky cross-sections. Regarding columns with slender cross-sections, the CSM approach demonstrated to provide similar results to the codified effective width method, but being a simpler design process. The reliability of the proposed approach has been finally demonstrated through the corresponding statistical analyses, and the best results were observed for the buckling curve codified in EN 1993-1-4 ^[1].

Given the purely analytic nature of the proposed approach, it can be used for several materials, such as stainless steel, carbon steel and aluminium, and cross-section types for which the CSM has been defined. Finally, the better estimation of the flexural buckling behaviour of columns will have a direct impact on the accuracy of beam-column checks, since the adoption of accurate end points is as important as having an adequate interaction expression. However, further research

is needed in order to assess the applicability of the proposed method to beam-columns, and also to extend the formulation to other cross-section shapes and materials.

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