The continuous strength method for structural stainless steel design
Outline:

- Current design basis of section design
- Continuous strength method (CSM)
- Comparisons with test results
- Reliability analysis
- Conclusions
Current basis for section design
Cross-section classification

Local buckling is considered through the process of cross-section classification:

• Basis of design in current stainless steel design codes (analogous to structural steel)
• Cross-sections placed into discrete behavioural classes
• Defines cross-section resistance and ductility
• Based on bi-linear (elastic, perfectly-plastic) material response
Cross-section classification

Four classes of cross-section:

- Class 1
- Class 2
- Class 3
- Class 4

Moment:
- $M_{pl}$
- $M_{el}$

Deformation:
Idealised bending stress distributions:

Class 1 and 2

Class 3

Class 4

Loss of effectiveness due to local buckling
Compression resistance

81 Compression stub column test results

Cross-section slenderness $\lambda_{cs}$

Class 1-3

Class 4

$A f_y$

$A_{eff} f_y$
Bending resistance

Bending test results

Class 1 and Class 2

Class 3

Class 4
The continuous strength method
Continuous strength method

Continuous strength method (CSM):

• Deformation based design approach allowing for strain hardening (and element interaction)

• Two key components to method:

1. Cross-section deformation capacity defined by the ‘Design base curve’; no more section classification

2. Material nonlinearity defined through a simple elastic, linear hardening material model

• Resistance derived from basic mechanics
Aims of method:

- More efficient and rational design
- Less scatter in prediction
- Minimum increase in complexity
- Providing engineer with more information
Design base curve

- Continuous relationship between cross-section deformation capacity and cross-section slenderness

**Base curve development:**

1. Cross-section slenderness definition
2. Slender and non-slender cross-section limit
3. Cross-section deformation capacity
   - Stub column tests and
   - Beam tests
Design base curve: Cross-section slenderness

- Cross-section slenderness definition: for cross-sections consisting of interconnected plate elements

\[ \lambda_{cs} = \sqrt{\frac{f_y}{\sigma_{cr,cs}}} \]

- Elastic buckling stress of the full cross-section → allowing for element interaction
- Elastic buckling stress of the most slender plate in the section
Design base curve: Slender and non-slender cross-section limit

- Maximum cross-section slenderness = 0.68
- Limit specified based on:
  1. \( \frac{N_{u,\text{test}}}{A\sigma_{0.2}} \) versus cross-section slenderness for stainless steel, carbon steel and aluminium alloys
  2. Current codified slenderness limits

\[ \lambda_{\text{cs,max}} = 0.68 \]
Cross-section deformation capacity definition

\[ \varepsilon_{csm} = \frac{\varepsilon_{lb} - 0.002}{\varepsilon_y} = \frac{\delta_u/L - 0.002}{\varepsilon_y} \quad \text{for } N_u \geq N_y \text{ and } \lambda_p \leq 0.68 \]

\[ \varepsilon_{csm} = \frac{\varepsilon_{lb} - 0.002}{\varepsilon_y} = \frac{\kappa_u y_{max} - 0.002}{\kappa_{el} y_{max}} \quad \text{for } M_u \geq M_{el} \text{ and } \lambda_p \leq 0.68 \]

\[ \varepsilon_{csm} = \frac{N_u}{N_y} \quad \text{for } N_u < N_y \]

\[ \varepsilon_{csm} = \frac{M_u}{M_{el}} \quad \text{for } M_u < M_{el} \]

Stub column tests

Beam tests

\[ \varepsilon_{lb} = \kappa_u y_{max} \]

\[ \varepsilon_{csm} = \varepsilon_{lb} - 0.002 \]
CSM ‘base curve’ defines deformation capacity – slenderness relationship.

\[
\frac{\varepsilon_{csm}}{\varepsilon_y} = \frac{0.25}{\lambda_{cs}^{3.6}} \quad \text{but} \quad \frac{\varepsilon_{csm}}{\varepsilon_y} \leq \min \left(15, \frac{0.1\varepsilon_u}{\varepsilon_y}\right)
\]

Derived from stub column tests
Material modelling

Material model chosen to reflect material stress-strain response:

\[ E_{sh} = \frac{f_u - f_y}{0.16\varepsilon_u - (\varepsilon_y + 0.002)} \]
Cross-section compression resistance

1. Determine cross-section slenderness for full section or most slender element

\[ \bar{\lambda}_{cs} = \sqrt{\frac{f_y}{\sigma_{cr,cs}}} \]

2. Obtain corresponding deformation capacity

\[ \frac{\varepsilon_{cs}}{\varepsilon_y} = \frac{0.25}{\bar{\lambda}_{cs}^{3.6}} \]

but \( \frac{\varepsilon_{cs}}{\varepsilon_y} \leq \min (15, 0.1 \frac{\varepsilon_u}{\varepsilon_y}) \)

3. Determine resulting local buckling stress \( f_{cs} \) from material model

\[ f_{cs} = f_y + E_{sh} \varepsilon_y \left( \frac{\varepsilon_{cs}}{\varepsilon_y} - 1 \right) \]

4. Section compression capacity is product of local buckling stress \( f_{cs} \) and gross cross-section area \( A \).

\[ N_{c,Rd} = N_{cs,Rd} = \frac{A f_{cs}}{\gamma_{M0}} \]
In bending, deformation capacity $\varepsilon_{\text{cs}}$ used to define outer fibre strain limit:

(a) Cross-section
(b) Strain
(c) Stress
Bending resistance determined from basic mechanics:

\[ M_{cs} = \int_A f_{cs} y \, dA \]

And may be approximated by:

\[
\frac{M_{cs}}{M_{pl}} = 1 + \frac{E_{sh}}{E} \left( \frac{W_{el}}{W_{pl}} \left( \frac{\varepsilon_{cs}}{\varepsilon_y} - 1 \right) \right) - \left( 1 - \frac{W_{el}}{W_{pl}} \right) \left( \frac{\varepsilon_{cs}}{\varepsilon_y} \right)^{-2}
\]
Cross-section bending resistance

1. Determine cross-section slenderness for full section or most slender element
   \[ \bar{\lambda}_{cs} = \sqrt{\frac{f_y}{\sigma_{cr,cs}}} \]

2. Obtain corresponding deformation capacity
   \[ \frac{\varepsilon_{cs}}{\varepsilon_y} = \frac{0.25}{\lambda_{cs}^{3.6}} \]
   but \( \frac{\varepsilon_{cs}}{\varepsilon_y} \leq \min(15, 0.1 \frac{\varepsilon_u}{\varepsilon_y}) \)

\[
M_{y,c,Rd} = M_{y,cs,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} \left[ 1 + \frac{E_{sh}}{E} \frac{W_{el,y}}{W_{pl,y}} \left( \frac{\varepsilon_{cs}}{\varepsilon_y} - 1 \right) - \left( 1 - \frac{W_{el,y}}{W_{pl,y}} \right) \right] \left( \frac{\varepsilon_{cs}}{\varepsilon_y} \right)^2 \]

Comparison with stub column tests data

- **No. of tests:** 81

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<tr>
<th></th>
<th>$N_{test}/N_{EC3}$</th>
<th>$N_{test}/N_{csm}$</th>
<th>$N_{csm}/N_{EC3}$</th>
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<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>1.222</td>
<td>1.088</td>
<td>1.123</td>
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<tr>
<td><strong>COV</strong></td>
<td>0.082</td>
<td>0.069</td>
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Comparison with beam tests data

No. of tests: 65

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<th>$M_{test}/M_{csm}$</th>
<th>$M_{csm}/M_{EC3}$</th>
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<tr>
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<td>1.134</td>
<td>1.191</td>
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<tr>
<td>COV</td>
<td>0.098</td>
<td>0.085</td>
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Reliability analysis

- Standard reliability analysis in accordance with EN 1990 – Annex D

<table>
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<tr>
<th>Test data</th>
<th>n</th>
<th>kd,n</th>
<th>b</th>
<th>Vδ</th>
<th>Vr</th>
<th>γM0</th>
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<tbody>
<tr>
<td>Stub columns</td>
<td>81</td>
<td>3.215</td>
<td>1.075</td>
<td>0.068</td>
<td>0.102</td>
<td>0.96</td>
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<tr>
<td>Beams</td>
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<td>1.108</td>
<td>0.086</td>
<td>0.114</td>
<td>0.98</td>
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</table>

![Graph 1](image1.png)

![Graph 2](image2.png)
Conclusions

Current approach

- Section classification underpins design of metallic structures
- Useful but artificial simplifications
  - Discrete behavioural classes; EPP $\sigma$-$\varepsilon$ curve

CSM

- Deformation based design
- Rational exploitation of strain hardening
- Enhanced structural efficiency
- EC3 and AISC approval underway
- Finalised version now recommended for others (researchers, codes)
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