DESIGN EXAMPLE 8 – RESISTANCE TO CONCENTRATED LOADS

An existing plate girder, previously subjected to an evenly distributed load, will be refurbished and will be subjected to a concentrated load. Check if the girder can resist the new load applied through a 12 mm thick plate. The girder is a simply supported I-section with a span according to the figure below. The top flange is laterally restrained.

Material grade 1.4462, hot rolled.

\[ f_y = 460 \text{ N/mm}^2 \]
\[ E = 200,000 \text{ N/mm}^2 \]

Flanges: \( 12 \times 200 \text{ mm}^2 \)
Web: \( 4 \times 500 \text{ mm}^2 \)
Stiffeners: \( 12 \times 98 \text{ mm}^2 \)
Weld throat thickness: \( 4 \text{ mm} \)

Structural analysis

Maximum shear and bending moment are obtained as

\[ V_{Ed} = \frac{F_{Ed}}{2} = \frac{110}{2} = 55 \text{ kN} \]
\[ M_{Ed} = \frac{F_{Ed}L}{4} = \frac{110 \times 2.5}{4} = 68.75 \text{ kNm} \]

Partial safety factors

\[ \gamma_{M0} = 1,1 \]
\[ \gamma_{M1} = 1,1 \]

Classification of the cross-section

\[ e = \sqrt[3]{\frac{235 \times 200}{460 \times 210}} = 0,698 \]
Web, subject to bending

\[ \frac{c}{t e} = \frac{500 - 2 \times \sqrt{2} \times 4}{4 \times 0.698} = 175 > 74.8, \text{ therefore the web is Class 4} \]

Flange, subject to compression

\[ \frac{c}{t e} = \frac{200 - 4 - 2 \times \sqrt{2} \times 4}{2 \times 12 \times 0.698} = 11.0 \leq 11.0, \text{ therefore the compression flange is Class 3} \]

Thus, overall classification of cross-section is Class 4

### Resistance to concentrated force

The design load should not exceed the design resistance, i.e.

\[ F_{Rd} = f_{yw} L_{eff} t / \gamma M_{1} \]

The effective length \( L_{eff} \) is given by

\[ L_{eff} = \lambda F l_y \]

where the reduction function is

\[ \lambda F = 0.5 \leq 1.0 \]

with the slenderness given by

\[ \lambda F = \sqrt[2]{\frac{l_y t_w f_{yw}}{F_{cr}}} \]

The effective loaded length is given by

\[ l_y = s_s + 2 t_f \left(1 + \sqrt{m_1 + m_2}\right) \]

Where

- \( s_s \) is the length of the stiff bearing and \( m_1 \) and \( m_2 \) are dimensionless parameters.

\[ m_1 = \frac{f_{yt} b_f}{f_{yw} t_w} \]

\[ m_2 = 0.02 \left(\frac{h_w}{t_f}\right)^2 \text{ for } \lambda F > 0.5 \]

\[ m_2 = 0 \text{ for } \lambda F \leq 0.5 \]

\( s_s \) is conservatively taken as twice the thickness of the load bearing plate, i.e. 24 mm.

\[ m_1 = \frac{460 \times 200}{460 \times 4} = 50 \]
m_2 = 0.02 \times \left[ \frac{500}{12} \right]^2 = 34.72, \text{ assuming } \bar{\lambda}_F > 0.5
\]

l_y = 24 + 2 \times 12 \times \left[ 1 + \sqrt{50 + 34.72} \right] = 268.90 \text{ mm}

The critical load is obtained as

\[ F_{cr} = 0.9 k_F E \frac{A_w^2}{h_w} \]

where the buckling coefficient is given by the load situation, type a.

k_f = 6 + 2 \left[ \frac{h_w}{a} \right]^2
\]

= 6 + 2 \times \left[ \frac{500}{2500} \right]^2 = 6.08

F_{cr} = 0.9 \times 6.08 \times 200000 \times \frac{4^3}{500} = 140.08 \text{ kN}

\[ \bar{\lambda}_F = \sqrt{\frac{268.90 \times 4 \times 460}{140.08 \times 10^3}} = 1.88 > 0.5, \text{ assumption OK} \]

\[ \chi_F = \frac{0.5}{1.88} = 0.266 \leq 1.0, \text{ OK} \]

L_{eff} = 0.266 \times 268.90 = 71.53 \text{ mm}

F_{Ed} = 110 \leq 460 \times 71.53 \times 4 / (1.1 \times 10^3) = 119.65 \text{ kN}

Hence the resistance exceeds the load.

**Interaction between transverse force, bending moment and axial force**

Interaction between concentrated load and bending moment is checked according to prEN 1993-1-5:2004.

0.8 \times \eta_1 + \eta_2 \leq 1.4

Where

\[ \eta_1 = \frac{N_{Ed}}{f_y \cdot A_{eff} / \gamma_M} + \frac{M_{Ed} + N_{Ed} e_N}{f_y \cdot W_{eff} / \gamma_M} \leq 1.0 \]
Calculation of effective cross-section properties
The flanges are Class 3 and hence fully effective.

The depth of the web has to be reduced with the reduction factor \( \rho \), welded web.

\[
\rho = \frac{0.772}{\bar{\lambda}_p} - \frac{0.125}{\bar{\lambda}_p} \leq 1
\]

\[
\bar{\lambda}_p = \frac{b/t}{28.4 \sqrt{k_{\sigma}}} \quad \text{where } b = d = 500 - 2 \times 4 \times \sqrt{2} = 488.68 \text{ mm}
\]

Assuming linearly varying symmetric stress distribution within the web,

\[
\psi = \frac{\sigma_2}{\sigma_1} = -1
\]

\[
\Rightarrow k_{\sigma} = 23.9
\]

\[
\bar{\lambda}_p = \frac{488.68/4}{28.4 \times 0.698 \times \sqrt{23.9}} = 1.26
\]

\[
\rho = \frac{0.772}{1.26} - \frac{0.125}{1.26^2} = 0.534 \leq 1
\]

\[
b_{\text{eff}} = \rho b_c = \rho \frac{b}{(1-\psi)} = 0.534 \times 488.68/(1-(-1)) = 130.48
\]

\[
b_{e1} = 0.4b_{\text{eff}} = 0.4 \times 130.48 = 52.19 \text{ mm}
\]

\[
b_{e2} = 0.6b_{\text{eff}} = 0.6 \times 130.48 = 78.29 \text{ mm}
\]

Calculate effective section modulus under bending
\( e_1 \) is taken as positive from the centroid of the upper flange and downwards

\[
A_{\text{eff}} = \sum_i A_i = b_1 t_f \times 2 + b_{e1} t_w + b_{e2} t_w + (h_w/2)t_w = 6321.92 \text{ mm}^2
\]

\[
e_{\text{eff}} = \frac{1}{A_{\text{eff}}} \sum_i A_i e_i = \frac{1}{A_{\text{eff}}} \left[ b_1 t_f (0) + b_1 t_1 (h_w + t_f) \right] + \left[ b_{e1} t_w (0.5(b_{e1} + t_f)) + b_{e2} t_w (0.5(h_w + t_f) - b_{e2}/2) + (b_{e2} / 2) t_w (0.75 h_w + 0.5 t_f) \right]
\]

\[
= 266.44 \text{ mm}
\]

\[
I_{\text{eff}} = \sum_i I_i + \sum_i A_i(e_{\text{eff}} - e_i)^2 = 2 \times \frac{b_1 t_f^3}{12} + \frac{t_w b_{e1}^3}{12} + \frac{t_w b_{e2}^3}{12} + \frac{t_w (h_w/2)^3}{12}
\]

\[
+ b_1 t_f \left( e_{\text{eff}} - 0 \right)^2 + b_1 t_f \left[ e_{\text{eff}} - (h_w + t_f) \right]^2 + b_{e1} t_w \left[ e_{\text{eff}} - 0.5(b_{e1} + t_f) \right]^2
\]

\[
+ b_{e2} t_w \left[ e_{\text{eff}} - 0.5(h_w + t_f) + b_{e2}/2 \right]^2 + (h_w/2)t_w \left[ e_{\text{eff}} - (0.75 h_w + 0.5 t_f) \right]^2
\]

\[
= 3.459 \times 10^8 \text{ mm}^4
\]
\[ W_{\text{eff}} = \frac{I_{\text{eff}}}{e_{\text{eff}} + 0.5t_f} = 1.270 \times 10^6 \text{mm}^3 \]

\[ \eta_1 = \frac{68.75 \times 10^6}{460 \times 1.270 \times 10^6 / 1.1} = 0.129 \]

\[ \eta_2 = \frac{110}{119.63} = 0.920 \]

\[ 0.8 \times \eta_1 + \eta_2 = 0.8 \times 0.129 + 0.920 = 1.023 < 1.4 \]

Therefore, the resistance of the girder to interaction between concentrated load and bending moment is adequate.

**Shear resistance**

The shear buckling resistance requires checking when \( h_w / t_w \geq \frac{52}{\eta} \varepsilon \) for unstiffened webs.

\[ h_w / t_w = \frac{500}{4} = 125 \geq \frac{52}{1.2} \times 0.698 = 30.2 \]

Therefore the shear buckling resistance has to be checked. It is obtained as

\[ V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \leq \frac{\eta f_{yw} h_w t}{\sqrt{3} \gamma_{M1}} \]

\[ V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t_w}{\gamma_{M1} \sqrt{3}} \]

\[ \chi_w = \eta = 1.2 \quad \text{for } \frac{h_w}{t_w} \leq 0.60 / \eta = 0.5 \]

\[ \chi_w = 0.11 + \frac{0.64}{\chi_w} - \frac{0.05}{\chi_w^2} \quad \text{for } \frac{h_w}{t_w} > 0.50 \]

\[ \chi_w = \left( \frac{500}{86.4 t_w} \right) = \left( \frac{500}{86.4 \times 4 \times 0.698} \right) = 2.072 > 0.5 \]

\[ \chi_w = 0.11 + \frac{0.64}{2.072} - \frac{0.05}{2.072^2} = 0.407 \]

The contribution from the flanges may be utilised if the flanges are not fully utilised to withstand the bending moment. However, the contribution is small and is conservatively not taken into account, i.e. \( V_{bf,Rd} = 0 \).

The shear buckling resistance can be calculated as:

\[ V_{bw,Rd} = \frac{0.407 \times 460 \times 500 \times 4}{1.1 \times \sqrt{3}} = 196.53 \text{ kN} > V_{td} = 55 \text{ kN} \]

The shear resistance of the girder is thus adequate.
Interaction between shear and bending

If \( \eta_3 \) does not exceed 0.5, the resistance to bending moment and axial force does not need to be reduced to allow for shear

\[
\eta_3 = \frac{V_{Ed}}{V_{bw,Rd}} \leq 1.0
\]

\[
= \frac{55}{196.55} = 0.280 \leq 0.5, \text{ therefore interaction need not to be considered.}
\]

Concluding remarks
The resistance of the girder exceeds the load imposed. Note that the vertical stiffeners at supports have not been checked. It should be done according to the procedure used in Design Example 7.